Module-03

Probability distributions

Let E be the event then the probability of an event 'E' is defined as

$$P(E) = \frac{\text{No. of favourable cases}}{\text{No. of possible cases}}$$

Random Variable: In a random experiment if a real variable is associated with every outcome then it is called random variable.

ex①: while tossing a coin, suppose that the value 1 is associated for the outcome 'head' and 0 for the outcome 'tail'. we have the sample space $S = \{H, T\}$ and if X is the random variable then $X(H) = 1$ and $X(T) = 0$

Range of $X = \{0, 1\}$

ex②: Suppose a coin is tossed twice we shall associate two different random variables X, Y as follows.

we have sample space

$S = \{HH, HT, TH, TT\}$

X = No. of heads in the outcomes

The association of elements in S to X as follows

outcome	HH	HT	TH	TT
Random Variable X	2	1	1	0

Range of $X = \{0, 1, 2\}$

Suppose Y = No. of tails in the outcome

outcome	HH	HT	TH	TT
Random Variable X	0	1	1	2

Range of $Y = \{0, 1, 2\}$

Type of Random Variable:-

① Discrete random Variable
② Contineous random Variable

① Discrete random Variable: If a random Variable takes finite no. of values (or) countably infinite no. of values then it is called discrete random Variable

ex: ① Tossing a coin and observing the outcome

② Tossing coins and observing the number of heads turning up.

③ Throwing a die & observing the numbers on the face.

② Contineous random Variable:-
If a random Variable takes infinite number of Values then it is called Contineous random Variable

ex ①: weight of articles

② Length of nails produced by machine

Probability function:-

If for each value of x_i of a discrete random variable 'X', we assign a real number $p(x_i)$ ∋;

(i) $p(x_i) \geq 0$

(ii) $\sum p(x_i) = 1$

Mean and Variance for general frequency distribution

$$(\text{Mean})\ \bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

$$\text{Variance}\ (\sigma^2) = \frac{\sum (x_i - \bar{x})^2 f_i}{\sum f_i}$$

Mean and Variance for discrete Probability distribution:-

$$\text{mean}\ (\mu) = \sum x_i p(x_i)$$

$$\text{Variance}\ (V) = \sum (x_i - \mu)^2 p(x_i)$$

or

$$\text{Variance}\ (V) = \sum x_i^2 p(x_i) - \mu^2$$

$$\text{standard deviation}\ (\sigma) = \sqrt{\text{variance}}$$

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NOTE ①: $p+q=1$

where $p \rightarrow$ probability of Success

$q \rightarrow$ probability of failure

NOTE ②: $P(A)+P(\bar{A})=1$

$P(A)=1-P(\bar{A})$

where $\bar{A}$ is the Complement of A.

Problems

① S.T the following distribution represents a discrete probability distribution, find the mean and Variance

x	10	20	30	40
$p(x)$	1/8	3/8	3/8	1/8

Soln: To prove the discrete probability distribution we have to verify the two conditions

① $p(x) \geq 0$

② $\Sigma p(x)=1$

(i) $p(x) \geq 0$, first condition is satisfied

(ii) $\frac{1}{8}+\frac{3}{8}+\frac{3}{8}+\frac{1}{8}=\frac{1+3+3+1}{8}=\frac{8}{8}$

$=1$

$\therefore \Sigma P(x)=1$

we observed that both the Conditions are satisfied

$\therefore$ given distribution represents a discrete probability distribution

$$\text{mean } (\mu) = \sum x_i \, P(x_i)$$

$$= 10\left(\tfrac{1}{8}\right) + 20\left(\tfrac{3}{8}\right) + 30\left(\tfrac{3}{8}\right) + 40\left(\tfrac{1}{8}\right)$$

$$= \frac{10}{8} + \frac{60}{8} + \frac{90}{8} + \frac{40}{8}$$

$$= \frac{200}{8}$$

$$\mu = 25$$

$$\text{Variance } (V) = \sum (x_i - \mu)^2 P(x_i)$$

$$= (10-25)^2 \frac{1}{8} + (20-25)^2 \frac{3}{8} + (30-25)^2 \frac{3}{8} + (40-25)^2 \frac{1}{8}$$

$$= \frac{225}{8} + 25 \times \frac{3}{8} + 25 \times \frac{3}{8} + 225 \times \frac{1}{8}$$

$$= \frac{225 + 75 + 75 + 225}{8}$$

$$= \frac{600}{8}$$

$$= 75$$

$$S.D\,(\sigma) = \sqrt{V}$$

$$= \sqrt{75}$$

(2) Find the value of K such that the following distribution represents a finite probability distribution and also find $P(x \leq 1)$, $P(x > 1)$ and $P(-1 < x \leq 2)$

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x	-3	-2	-1	0	1	2	3
$p(x)$	K	2K	3K	4K	3K	2K	K

Solⁿ: we must have (i) $p(x) \geq 0$

(ii) $\sum P(x) = 1$

The first condition is satisfied if $K \geq 0$, we have to find $K \ni; \sum P(x) = 1$

$\sum P(x) = K + 2K + 3K + 4K + 3K + 2K + K = 16K$

$\sum P(x) = 1$

$\Rightarrow \quad 16K = 1$

$K = \frac{1}{16}$,

$\therefore \quad K = \frac{1}{16} > 0$

$(40-25)^2 \frac{1}{8}$

The discrete / finite probability distribution is

x	-3	-2	-1	0	1	2	3
$p(x)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

mean $\mu = \sum x_i P(x_i)$

$= \frac{1}{16}(-3) + \frac{2}{16}(-2) + \frac{3}{16}(-1) + \frac{4}{16}(0) +$

$\frac{3}{16}(1) + \frac{2}{16}(2) + \frac{1}{16}(3)$

$= \frac{1}{16}(\cancel{-3} - \cancel{4} - \cancel{3} + 0 + \cancel{3} + \cancel{4} + \cancel{3})$

$= \frac{1}{16}(0)$

$\boxed{\mu = 0}$

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Variance $V = \sum (x_i - \mu)^2 p(x_i)$

$$V = (-3-0)^2 \frac{1}{16} + (-2-0)^2 \frac{2}{16} + (-1-0)^2 \frac{3}{16}$$
$$+ (0-0)^2 \frac{4}{16} + (1-0)^2 \frac{3}{16} + (2-0)^2 \frac{2}{16}$$
$$+ (3-0)^2 \frac{1}{16}$$

$$= \frac{9\times 1}{16} + \frac{4\times 2}{16} + \frac{1\times 3}{16} + \frac{0\times 4}{16} + \frac{1\times 3}{16}$$
$$+ \frac{4\times 2}{16} + \frac{9\times 1}{16}$$

$$= \frac{1}{16}(9+8+3+0+3+8+9)$$

$$= \frac{11}{16}(40)$$

$V = \frac{5}{2}$ // $\quad$ S.D $(\sigma) = \sqrt{V} = \sqrt{5/2}$

Thus $K = \frac{1}{16}$, $\mu = 0$, $V = \frac{5}{2}$, $\sigma = \sqrt{5/2}$

now we have to find $P(x) \leq 1$

$$P(x \leq 1) = P(-3) + P(-2) + P(-1) + P(0) + P(1)$$
$$= \frac{1}{16} + \frac{2}{16} + \frac{3}{16} + \frac{4}{16} + \frac{3}{16}$$
$$= \frac{1}{16}(1+2+3+4+3)$$

$P(x \leq 1) = \frac{13}{16}$ //

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(ii) $P(x>1) = P(2)+P(3)$

$$= \frac{2}{16} + \frac{1}{16}$$

$$= \frac{1}{16}(2+1)$$

$$= \frac{3}{16}$$

(iii) $P(-1 < x \leq 2) = P(0)+P(1)+P(2)$

$$= \frac{4}{16} + \frac{3}{16} + \frac{2}{16}$$

$$= \frac{4+3+2}{16}$$

$$P(-1 < x \leq 2) = \frac{9}{16}$$

(3) The probability distribution function of a variate X is given by the following table

x	0	1	2	3	4	5	6
p(x)	K	3K	5K	7K	9K	11K	13K

For what value of K this represents a valid probability distribution? also find $P(x \geq 5)$ and $P(3 < x \leq 6)$.

Sol^n: The probability distribution is valid if (i) $P(x) \geq 0$ and

(ii) $\Sigma P(x) = 1$

hence we must have $K \geq 0$ and

$$\Sigma P(x) = 1$$

$$K + 3K + 5K + 7K + 9K + 11K + 13K = 1$$

$$49K = 1$$

$$K = \frac{1}{49}$$

∴ probability distribution function is valid

for $K = \frac{1}{49}$

Now we have to find $p(x \geq 5)$ & $p(3 < x \leq 6)$

$p(x \geq 5) = P(5) + P(6)$

$= 11K + 13K$

$= 24K$

$= 24 \times \frac{1}{49}$

$= \frac{24}{49}$

$P(3 < x \leq 6) = P(4) + P(5) + P(6)$

$= 9K + 11K + 13K$

$= 33K$

$= 33 \times \frac{1}{49}$

$P(3 < x \leq 6) = \frac{33}{49}$

(4) The probability distribution of a finite random Variable X is given by the following table

x	-2	-1	0	1	2	3
$P(x)$	0.1	K	0.2	2K	0.3	K

Find the value of K, mean & Variance

Solⁿ: we must have $p(x) \geq 0$ and $\sum p(x) = 1$ for a probability distribution

we have to find K ∋;

$\sum P(x) = 1$

$0.1 + K + 0.2 + 2K + 0.3 + K = 1$

$4K + 0.6 = 1$

$4K = 1 - 0.6$

$4K = 0.4$

$K = 0.1$

The probability distribution of a finite random variable X is

x	-2	-1	0	1	2	3
$P(x)$	0.1	0.1	0.2	0.2	0.3	0.1

mean $\mu = \sum x_i p(x_i)$

$= -2(0.1) + (-1)(0.1) + 0(0.2) + 1(0.2) + 2(0.3) + 3(0.1)$

$= \cancel{-0.2} - 0.1 + 0 + \cancel{0.2} + 0.6 + 0.3$

$\mu = 0.8$

Variance $(V) = \sum (x_i - \mu)^2 p(x_i)$

$= (-2-0.8)^2(0.1) + (-1-0.8)^2(0.1) + (0-0.8)^2(0.2) + (1-0.8)^2(0.2) + (2-0.8)^2(0.3) + (3-0.8)^2(0.1)$

$= (-2.8)^2(0.1) + (-1.8)^2(0.1) + (-0.8)^2(0.2) + (0.2)^2(0.2) + (1.2)^2(0.3) + (2.2)^2(0.1)$

$= 7.84 \times 0.1 + 3.24 \times 0.1 + 0.64 \times 0.2 + 0.04 \times 0.2 + 1.44 \times 0.2 + 4.84 \times 0.1$

$= 0.784 + 0.324 + 0.128 + 0.008 + 0.432 + 0.484$

$V = 2.16$

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(5) A random variable X has the following probability function for various values of x

x	0	1	2	3	4	5	6	7
$p(x)$	0	K	2K	2k	3K	K^2	$2K^2$	$7K^2+K$

(i) Find K

(ii) Evaluate $P(x<6)$, $P(x \geq 6)$ and $P(3 < x \leq 6)$

also find the probability distribution and distribution function of x.

Solⁿ: To find the probability distribution we must have $p(x) \geq 0$ and $\Sigma p(x) = 1$

The first condition is satisfied for $K \geq 0$, we have to find K ∋;

$\Sigma p(x) = 1$

$0 + K + 2k + 2k + 3K + K^2 + 2K^2 + 7K^2 + K = 1$

$10K^2 + 9K - 1 = 0$

$10K^2 + 10K - K - 1 = 0$

$10K(K+1) - 1(K+1) = 0$

$(K+1)(10K+1) = 0$

$K+1 = 0$ or $10K - 1 = 0$

$K = -1$ or $10K = 1$

$K = \frac{1}{10}$

If $K = -1$, first condition fails

So $K \neq -1$

∴ Take $K = \frac{1}{10} = 0.1$

$K = 0.1$

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hence table (or) probability distribution is

x	0	1	2	3	4	5	6	7
$p(x)$	0	0.1	0.2	0.2	0.3	0.01	0.02	0.17

(ii) $p(x < 6)$

$= P(0) + P(1) + P(2) + P(3) + P(4) + P(5)$

$= 0 + 0.1 + 0.2 + 0.2 + 0.3 + 0.01$

$p(x<6) = 0.81$

$p(x \geq 6) = p(6) + p(7)$

$= 0.02 + 0.17$

$= 0.19$

$P(3 < x \leq 6) = P(4) + P(5) + P(6)$

$= 0.3 + 0.01 + 0.02$

$= 0.33$

distribution function of x is

x	0	1	2	3	4	5	6	7
$f(x)$	0	$0+0.1 = 0.1$	$0.1+0.2 = 0.3$	$0.3+0.2 = 0.5$	$0.5+0.3 = 0.8$	$0.8+0.01 = 0.81$	$0.81+0.02=0.83$	$0.83+0.17 = 1$

(6) A random variable X take the values $-3, -2, -1, 0, 1, 2, 3$ such that $P(X=0) = P(X<0)$ and $P(X=-3) = P(X=-2) = P(X=-1) = P(X=1) = P(X=2) = P(X=3)$. find the probability distribution.

Sol^n:

X	-3	-2	-1	0	1	2	3
P(X)	P_1	P_2	P_3	P_4	P_5	P_6	P_7

By data $P(X=0) = P(X<0)$

$\Rightarrow P(X=0) = P(X=-1) + P(X=-2) + P(X=-3)$

$$P_4 = P_3 + P_2 + P_1 \quad \text{--- (1)}$$

By data

$$P_1 = P_2 = P_3 = P_5 = P_6 = P_7 \quad \text{--- (2)}$$

we must have $\Sigma P(x) = 1$

$$P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7 = 1 \quad \text{--- (3)}$$

use (2) in (3)

$$P_1 + P_1 + P_1 + P_4 + P_1 + P_1 + P_1 = 1$$

$$6P_1 + P_4 = 1 \quad \text{--- (4)}$$

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use ③ in ①

eqⁿ ① becomes $P_4 = P_1 + P_1 + P_1$

$P_4 = 3P_1$

eqⁿ ④ becomes

$6P_1 + 3P_1 = 1$

$9P_1 = 1$

$\boxed{P_1 = \frac{1}{9}}$

hence $P_4 = 3P_1 = 3 \times \frac{1}{9} = \frac{1}{3}$

$\therefore \boxed{P_4 = \frac{1}{3}}$

Thus probability distribution is

X	-3	-2	-1	0	1	2	3
P(X)	1/9	1/9	1/9	1/3	1/9	1/9	1/9

7) If the random variable X take the values 1, 2, 3, 4 such that $2P(X=1) = 3P(X=2) = P(X=3) = 5P(X=4)$, find the probability distribution function of X.

Solⁿ:

X	1	2	3	4
P(X)	P_1	P_2	P_3	P_4

$2P_1 = 3P_2 = P_3 = 5P_4$ — ①

also we must have $\sum P(x) = 1$

$P_1 + P_2 + P_3 + P_4 = 1$ — ②

from ①

$2P_1 = 3P_2$, $2P_1 = P_3$, $2P_1 = 5P_4$

$P_2 = \frac{2}{3} P_1$ $P_3 = 2P_1$ $P_4 = \frac{2}{5} P_1$

hence ② becomes

$$P_1 + \frac{2}{3}P_1 + 2P_1 + \frac{2}{5}P_1 = 1$$

lcm is 15

$$\frac{15P_1 + 10P_1 + 30P_1 + 6P_1}{15} = 1$$

$$\frac{61}{15}P_1 = 1 \Rightarrow P_1 = \frac{15}{61}$$

hence we get $P_2 = \frac{2}{3} \times P_1 = \frac{2}{3} \times \frac{15}{61} = \frac{30}{3 \times 61} = \frac{10}{61}$

$$P_3 = 2P_1 = 2 \times \frac{15}{61} = \frac{30}{61}$$

$$P_4 = \frac{2}{5} P_1 = \frac{2}{5} \times \frac{15}{61} = \frac{6}{61}$$

The probability distribution function of X

X	1	2	3	4
P(X)	$15/61$	$10/61$	$30/61$	$6/61$
f(x)	$15/61$	$15/61 + 10/61 = 25/61$	$25/61 + 30/61 = 55/61$	$55/61 + 6/61 = 61/61 = 1$

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(8) If X is a discrete random variable having $p(x)$ defined as follows

$$p(x) = \begin{cases} x/15, & \text{if } 1 \leq x \leq 5 \\ 0 & \text{if } x > 5 \end{cases}$$

S.T $p(x)$ is a probability function and find $p(X=1 \text{ or } 2)$

Solⁿ: The probability distribution is as follows

x	1	2	3	4	5	6,7,8....
$p(x)$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$	0

we have ① $p(x) \geq 0$

② $\sum p(x) = 1$

$\therefore$ $p(x)$ is probability function

Now $p(X = 1 \text{ or } 2) = p(X=1) + p(X=2)$

$$= \frac{1}{15} + \frac{2}{15}$$

$$= \frac{3}{15}$$

$$= \frac{1}{5}$$

Binomial distribution

If p is the probability of success and q is the probability of failure, the probability of x success out of n trials is given by $p(x) = {}^nC_x \, p^x q^{n-x}$

We form the following probability distribution of $(x, p(x))$ where $x = 0, 1, 2 \cdots n$

x	0	1	2	n
$p(x)$	q^n	${}^nC_1 q^{n-1} p$	${}^nC_2 q^{n-2} p^2$	p^n

It may be observed that the value of $p(x)$ for different values $x = 0, 1, 2 \ldots n$ are the successive terms in the binomial expansion of $(q+p)^n$ and accordingly this distribution is called binomial distribution.

$\Sigma p(x) = q^n + {}^nC_1 q^{n-1} p + {}^nC_2 q^{n-2} p^2 + \cdots + p^n$
$= (q+p)^n$
$= 1^n$
$= 1$

hence $p(x)$ is a probability function

Mean & Standard deviation of Binomial distribution :-

(OR)

Derive Mean & S.D of Binomial distribution

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Mean, $\mu = \sum_{x=0}^{n} x p(x)$

W.K.T
$^nC_r = \frac{n!}{(n-r)!\,r!}$

$= \sum_{x=0}^{n} x \, ^nC_x \, p^x q^{n-x}$

$= \sum_{x=0}^{n} x \frac{n!}{(n-x)!\,x!} p^x q^{n-x}$

$= \sum_{x=0}^{n} \frac{\cancel{x}\, n(n-1)!}{(\cancel{x})(x-1)!\,(n-x)!} p^x q^{n-x}$

$= \sum_{x=0}^{n} \frac{n(n-1)!}{(x-1)!\,(n-x)!} p \cdot p^{x-1} q^{n-x}$

$= np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!\,\boxed{(n-1)-(x-1)}!} p^{x-1} q^{(n-1)-(x-1)}$

$= np \sum_{x=1}^{n} \, ^{(n-1)}C_{x-1} \, p^{x-1} q^{(n-1)-(x-1)}$

$\cancel{\frac{n!}{r!(n-r)!}} = {}^nC_r$

$= np (q+p)^{n-1}$

$= np(1)$

$\mu = np$ //

Variance (V) $= \sum_{x=0}^{n} x^2 p(x) - \mu^2$ — (*)

Now, $\sum_{x=0}^{n} x^2 p(x) = \sum_{x=0}^{n} [x(x-1) + x] p(x)$

$= \sum_{x=0}^{n} x(x-1) p(x) + x p(x)$

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$$= \sum_{x=0}^{n} x(x-1)\,p(x) + \sum_{x=0}^{n} x\,p(x)$$

$$= \sum_{x=0}^{n} x(x-1)\, {}^{n}C_{x}\, p^{x} q^{n-x} + np$$

$$= \sum_{x=0}^{n} x(x-1) \frac{n!}{(n-x)!\,x!} p^{x} q^{n-x} + np$$

$$= \sum_{x=0}^{n} \frac{\cancel{x(x-1)}\, n(n-1)(n-2)!}{(n-x)!\, \cancel{x(x-1)}\,[x-2]!} p^{2}\, p^{x-2} q^{n-x} + np$$

$$= \sum_{x=0}^{n} \frac{n(n-1)(n-2)!}{(x-2)!\,(n-x)!} p^{2}\, p^{x-2} q^{n-x} + np$$

$$= n(n-1)p^{2} \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!\,[(n-2)-(x-2)]!} p^{x-2} \cdot q^{(n-2)-(x-2)} + np$$

$$= n(n-1)p^{2} \sum_{x=2}^{n} {}^{(n-2)}C_{(x-2)}\, p^{x-2} q^{(n-2)-(x-2)} + np$$

$$= n(n-1)p^{2} (q+p)^{n-2} + np$$

$$= n(n-1)p^{2} (1)^{n-2} + np$$

$$\sum x^{2} p(x) = n(n-1)p^{2} + np$$

$$= (n^{2}-n)p^{2} + np$$

$$\sum x^{2} p(x) = n^{2}p^{2} - np^{2} + np \;//$$

(*) becomes

∴ Variance V

variance (V

S.D (σ) =

Thus for

$\mu = np$,

(1) Find

which

Soln: w.k

(*) becomes

$\therefore$ variance $V = (n^2p^2 - np^2 + np) - (np)^2$

$= \cancel{n^2p^2} - np^2 + np - \cancel{n^2p^2}$

$= -np^2 + np$

$= np(-p+1)$

$= np(1-p)$

$= npq$

variance $(V) = npq$

S.D $(\sigma) = \sqrt{V} = \sqrt{npq}$

W.K.T
$p+q=1$
$q=1-p$

Thus for binomial distribution
$\mu = np$, variance $(V) = npq$
S.D $(\sigma) = \sqrt{npq}$

Problems

① Find the binomial distribution which has mean 2 and variance 4/3

Solⁿ: W.K.T Mean $(\mu) = np$

Given $\mu = np = 2$

$\therefore np = 2$

Variance $(V) = npq = 4/3$

$npq = 4/3$

$2q = 4/3$

$q = 2/3$

$p+q=1 \Rightarrow p = 1-q = 1-2/3 = \frac{3-2}{3} = 1/3$

$\therefore p = 1/3, \quad q = 2/3$

$np = 2$

$n = 2/p = 2/\tfrac{1}{3}$

$\boxed{n = 6}$

$\therefore$ Binomial probability distribution

$P(x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$

$P(x) = {}^{6}C_{x} \left(\frac{1}{3}\right)^{x} \left(\frac{2}{3}\right)^{6-x}$

The distribution of probability is follows

x	0	1	2	3	4	5	6
$P(x)$	$\left(\frac{2}{3}\right)^{6}$	${}^{6}C_{1}(1/3)(2/3)^{5}$	${}^{6}C_{2}(1/3)^{2}(2/3)^{4}$	${}^{6}C_{3}(1/2)^{3}(2/3)^{3}$	${}^{6}C_{4}(1/2)^{4}(2/3)^{2}$	${}^{6}C_{5}(1/3)^{5}(2/3)$	${}^{6}C_{6}(1/2)^{6}(2/3)^{0}$

(2) when a coin is tossed 4 times find the probability of getting

1. exactly one head
2. atmost 3 heads
3. atleast 2 heads

Soln: $p = P(H) = \frac{1}{2}$

W.K.T $p + q = 1$

$q = 1 - p$

$= 1 - 1/2$

$= 1/2$

$\boxed{p = 0.5}$

$n = 4$

w.K.T

$$P(x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$$

$$= {}^{4}C_{x}\, (0.5)^{x}\, (0.5)^{4-x}$$

(i) $P(\text{exactly one head}) = P(x=1) = {}^{4}C_{1}(0.5)^{1}(0.5)^{4-1}$

$P(x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$

$= 4\,(0.5)(0.5)^{3}$

$= 0.25$ //

(ii) $P(\text{atmost 3 heads}) = P(x \leq 3)$

$P(x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$

$= P(x=0) + P(x=1) + P(x=2) + P(x=3)$

$= {}^{4}C_{0}(0.5)^{0}(0.5)^{4-0} + {}^{4}C_{1}(0.5)^{1}(0.5)^{4-1}$

$+ {}^{4}C_{2}(0.5)^{2}(0.5)^{4-2} + {}^{4}C_{3}(0.5)^{3}(0.5)^{4-3}$

$= (1)(1)(0.5)^{4} + 4(0.5)(0.5)^{3} + 6(0.5)^{2}(0.5)^{2} + 4(0.5)^{3}(0.5)$

$= 0.0625 + 0.25 + 0.375 + 0.25$

$= 0.9375$ //

(iii) $P(\text{atleast 2 heads}) = P(x \geq 2)$

$= P(x=2) + P(x=3) + P(x=4)$

$P(x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$

$= {}^{4}C_{2}(0.5)^{2}(0.5)^{4-2} + {}^{4}C_{3}(0.5)^{3}(0.5)^{4-3} + {}^{4}C_{4}(0.5)^{4}(0.5)^{4-4}$

$= 6(0.5)^{2}(0.5)^{2} + 4(0.5)^{3}(0.5) + (1)(0.5)^{4}(0.5)^{0}$

$= 6(0.5)^{4} + 4(0.5)^{4} + (1)(0.5)^{4}$

(1)

$= 0.375$

$= 0.687$

(3) The pro...
-ed by ...
1/10. If ...
what 4 ...
(i) Exa...
(ii) Atle...
(iii) None...

Sol^n:

(i) .
P(ex...

(ii) P(at...

$= 0.375 + 0.25 + 0.0625$

$= \underline{0.6875}$

(3) The probability that a pen manufactured by a factory be defective is $1/10$. If 12 such pens are manufactured what is the probability that

(i) Exactly two are defective

(ii) Atleast two are defective

(iii) None of them are defective

Sol^n: probability of a defective pen is $p = \frac{1}{10} = 0.1$

probability of non defective pen is $p + q = 1$

$q = 1 - p$

$q = 1 - 0.1$

$q = 0.9$

$n = 12$, we have $P(x) = {}^{n}C_{x}\, p^{x} q^{n-x}$

$P(x) = 12C_{x} (0.1)^{x} (0.9)^{12-x}$

(i).

P(exactly 2 are defective)

$= P(x = 2)$

$= 12C_{2} (0.1)^{2} (0.9)^{12-2}$

$= 66 (0.01)(0.9)^{10}$

$= 0.2301$

(ii) P(atleast two are defective) $= P(x \geq 2)$

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$= 1 - P(x < 2)$
$= 1 - [P(x=0) + P(x=1)]$
$= 1 - [12_{C_0} (0.1)^0 (0.9)^{12-0} + 12_{C_1} (0.1)^1 (0.9)^{12-1}]$
$= 1 - [(1)(1)(0.9)^{12} + 12(0.1)(0.9)^{11}]$
$= 1 - 0.659$
$= \underline{0.3409}$

(iii) P(None of them are defective) $= P(x=0)$
$= 12_{C_0} (0.1)^0 (0.9)^{12-0}$
$= (1)(1)(0.9)^{12}$
$= 0.2824$

(4) In a consigment of electric lamps 5% are defective. If a random sample of 8 lamps are inspected what is the probability that one or more lamps are defective?

Soln: probability of defective lamp

$p = 5\% = \frac{5}{100} = 0.05$

W.K.T $p + q = 1 \Rightarrow q = 1 - p \Rightarrow q = 1 - 0.05 = 0.95$

we have $n = 8$

& $p(x) = n_{C_x} p^x q^{n-x}$

$= 8_{C_x} (0.05)^x (0.95)^{8-x}$

where x denotes defective lamp

p(one or more lamps defective)

$= P(x \geq 1)$

$= 1 - P(x < 1)$

$= 1 - P(x=0)$

$= 1 - {}^8C_0 (0.05)^0 (0.95)^{8-0}$

$= 1 - (1)(1)(0.95)^8$

$= 1.06634$

$P(x \geq 1) = 0.33657$ //

(or) You can do like this you will get same answer as above

$P(x \geq 1) = P(x=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5) + P(x=6) + P(x=7) + P(x=8)$

$= {}^8C_1 (0.05) (0.95)^{8-1} + {}^8C_2 (0.05)^2 (0.95)^{8-2}$
$+ {}^8C_3 (0.05)^3 (0.95)^{8-3} + {}^8C_4 (0.05)^4 (0.95)^{8-4}$
$+ {}^8C_5 (0.05)^5 (0.95)^{8-5} + {}^8C_6 (0.05)^6 (0.95)^{8-6}$
$+ {}^8C_7 (0.05)^7 (0.95)^{8-7} + {}^8C_8 (0.05)^8 (0.95)^{8-8}$

$= 8(0.05)(0.95)^7 + 28(0.05)^2(0.95)^6 + 56(0.05)^3(0.95)^5 + 70(0.05)^4(0.95)^4$
$+ 56(0.05)^5(0.95)^3 + 28(0.05)^6(0.95)^2$
$+ 8(0.05)^7(0.95)^1 + (1)(0.05)^8 \underbrace{(0.95)^0}_{(1)}$

$= 0.33657$

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(5) The probability that a person aged 60 years will live upto 70 is 0.65 what is the probability that out of 10 persons aged 60 atleast 7 of them will live upto 70.

Sol: Let x be the number of persons aged 60 years living upto 70 years

For this we have

$$p = 0.65$$

$$p+q = 1 \Rightarrow q = 1-p = 1-0.65$$

$$q = 0.35$$

Now we have $n = 10$

& $p(x) = {}^{n}C_{x} p^{x} q^{n-x}$

∴ we have to find $p(x \geq 7)$

$$p(x \geq 7) = p(x=7) + p(x=8) + p(x=9) + p(x=10)$$

$$= {}^{10}C_{7}(0.65)^{7}(0.35)^{10-7} + {}^{10}C_{8}(0.65)^{8}(0.35)^{10-8}$$

$$+ {}^{10}C_{9}(0.65)^{9}(0.35)^{10-9} + {}^{10}C_{10}(0.65)^{10}(0.35)^{10-10}$$

$$= {}^{10}C_{7}(0.65)^{7}(0.35)^{3} + {}^{10}C_{8}(0.65)^{8}(0.35)^{2}$$

$$+ {}^{10}C_{9}(0.65)^{9}(0.35)^{1} + {}^{10}C_{10}(0.65)^{10}(0.35)^{0}$$

$$= 0.25221 + 0.17565 + 0.07249 + 0.01346$$

$$= 0.5138$$

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(OR) you can do like this also will get same answer

$$P(x \geq 7) = 1 - P(x < 7)$$

$$= 1 - [P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5) + P(x=6)]$$

$$= 0.5138$$

6. The number of telephone lines busy at an instant of time is a binomial variate with probability 0.1 that a line is busy. If 10 lines are chosen at random, what is the probability that
1. no line is busy
2. All lines are busy
3. Atleast one line is busy
4. Atmost two lines are busy.

Soln: Let x denote the number of telephone lines busy.

we have by data $p = 0.1$

$$p + q = 1$$

$$q = 1 - p = 1 - 0.1 = 0.9$$

$$q = 0.9$$

also $n = 10$

we have $p(x) = n_{C_x} \, p^x q^{n-x}$

$$p(x) = 10_{C_x} (0.1)^x (0.9)^{10-x}$$

(i) probability that no line is busy

$= P(x=0)$

$= {}^{10}C_0 (0.1)^0 (0.9)^{10-0}$

$= (1)(1)(0.9)^{10}$

$= 0.3487$

(ii) probability that all lines are busy

$= P(x=10)$

$= {}^{10}C_{10} (0.1)^{10} (0.9)^{10-10}$

$= (1)(0.1)^{10}(0.9)^0$

$= (1)(0.1)^{10}(1)$

$= (0.1)^{10}$

(iii) probability that atleast one line is busy $= P(x \geq 1)$

$= 1 - P(x<1)$

$= 1 - [P(x=0)]$

$= 1 - [{}^{10}C_0 (0.1)^0 (0.9)^{10}]$

$= 1 - [(1)(1)(0.9)^{10}]$

$= 1 - 0.34867$

$= 0.65132$

(iv) probability that atmost 2 lines are busy $= P(x \leq 2)$

$= P(x=0) + P(x=1) + P(x=2)$

$= {}^{10}C_0 (0.1)^0 (0.9)^{10-0} + {}^{10}C_1 (0.1)^1 (0.9)^{10-1}$

$+ {}^{10}C_2 (0.1)^2 (0.9)^{10-2}$

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$= (1)(1)(0.9)^{10} + 10(0.1)(0.9)^9 + 45(0.1)^2(0.9)^8$
$= 0.9298$

Do yourself

(7) In a quiz contest of answering 'Yes' or 'NO' what is the probability of guessing atleast 6 answers correctly out of 10 questions asked? Also find the probability of the same if there are 4 options for a correct answer.

Sol: Let x denotes the correct answer

$p = \frac{1}{2} = 0.5$

$p+q = 1 \Rightarrow q = 1-p = 1-0.5 = 0.5$

$q = 0.5$

$n = 10$

we have $P(x) = n_{C_x} p^x q^{n-x}$

$= 10_{C_x} (0.5)^x (0.5)^{10-x}$

$= 10_{C_x} (0.5)^x (0.5)^{10} (0.5)^{-x}$

$= 10_{C_x} (0.5)^{10} (0.5)^{x-x}$

$(0.5)^{x-x} = (0.5)^0 = 1$

$P(x) = 10_{C_x} (0.5)^{10}$

probability of getting atleast 6 answers correctly $= P(x \geq 6)$

$= P(x=6) + P(x=7) + P(x=8) + P(x=9) + P(x=10)$

$= 10_{C_6} (0.5)^{10} + 10_{C_7} (0.5)^{10} + 10_{C_8} (0.5)^{10} + 10_{C_9} (0.5)^{10} + 10_{C_{10}} (0.5)^{10}$

$$= (0.5)^{10}\left[{}^{10}C_6 + {}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10}\right]$$

$$= (0.5)^{10}\left[210 + 120 + 45 + 10 + 1\right]$$

$$P(x \geq 6) = \underline{\underline{0.3769}}$$

OR

$$P(x \geq 6) = 1 - P(x < 6)$$

$$= 1 - \left[P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5)\right]$$

$$= 1 - \left[{}^{10}C_0 (0.5)^{10} + {}^{10}C_1 (0.5)^{10} + {}^{10}C_2 (0.5)^{10} + {}^{10}C_3 (0.5)^{10} + {}^{10}C_4 (0.5)^{10} + {}^{10}C_5 (0.5)^{10}\right]$$

$$= 1 - (0.5)^{10}\left[{}^{10}C_0 + {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4 + {}^{10}C_5\right]$$

$$= 1 - (0.5)^{10}\left[1 + 10 + 45 + 120 + 210 + 252\right]$$

$$= \underline{\underline{0.3769}}$$

OR

$$P(x \geq 6) = P(x=6) + P(x=7) + P(x=8) + P(x=9) + P(x=10)$$

we have $P(x) = {}^{n}C_x\, p^x q^{n-x}$

$$= {}^{10}C_6 (0.5)^6 (0.5)^4 + {}^{10}C_7 (0.5)^7 (0.5)^3 + {}^{10}C_8 (0.5)^8 (0.5)^2 + {}^{10}C_9 (0.5)^9 (0.5)^1 + {}^{10}C_{10} (0.5)^{10} (0.5)^0$$

$= {}^{10}C_6 (0.5)^{10} + {}^{10}C_7 (0.5)^{10} + {}^{10}C_8 (0.5)^{10} + {}^{10}C_9 (0.5)^{10}$
$+ {}^{10}C_{10} (0.5)^{10}$

$= (0.5)^{10} [{}^{10}C_6 + {}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10}]$

$= (0.5)^{10}(386) = \underline{\underline{0.3769}}$

(8) In a sampling large number of parts manufactured by a company, the mean number of defectives in samples of 20 is 2, out of 1000 such samples how many would be expected to contain atleast 3 defective parts.

Sol^n: Given $\mu = np = 2$ by data $n = 20$

$\therefore np = 2 \Rightarrow 20p = 2 \Rightarrow p = \frac{2}{20} = \frac{1}{10}$

$p = 1/10 = 0.1$ W.K.T $p + q = 1$

$q = 1 - p = 1 - 0.1 = 0.9$

$q = 0.9$

let x denotes the defective part

$p(x \geq 3)$ = probability of atleast 3 defective parts

$p(x \geq 3) = P(x = 3) + P(x = 4) + P(x = 5) \ldots + P(x = 20)$

It is lengthy procedure

OR

$P(x \geq 3) = 1 - P(x < 3)$

$= 1 - [P(x = 0) + P(x = 1) + P(x = 2)]$

we have $p(x) = {}^nC_x p^x q^{n-x}$

$= 1 - [\, {}^{20}C_0 (0.1)^0 (0.9)^{20-0}$

$+ {}^{20}C_1 (0.1)^1 (0.9)^{20-1}$

$+ {}^{20}C_2 (0.1)^2 (0.9)^{20-2} \,]$

$= 1 - [\,20_{C_0} (0.9)^{20} + 20_{C_1} (0.9)^{19} (0.1)$

$+ 20_{C_2} (0.1)^2 (0.9)^{18}\,]$

$= 1 - [\,(1)(0.9)^{20} + 20\,(0.1)(0.9)^{19}$

$+ 190\,(0.1)^2 (0.9)^{18}\,]$

$p(x \geq 3) = 0.323$

Thus Number of defectives in 1000 Samply is 323//

(9) In 800 families with 5 children each how many family would be expected to have (i) 3 boys
(ii) 5 girls
(iii) either 2 or 3 boys
(iv) atmost 2 girls
by assuming the probabilities for boys and girls to be equal

Sol^n: p = probability of having a boy $= \frac{1}{2} = 0.5$
q = probability of having a girl $= \frac{1}{2} = 0.5$
let x denotes the no. of boys in the family

$n = 5$

$P(x) = n_{C_x}\, p^x q^{n-x}$

$P(x) = 5_{C_x}\, (0.5)^x (0.5)^{n-x}$

(i) $P(x = 3) = 5_{C_3}\, (0.5)^3 (0.5)^{5-3}$

$= 10\,(0.5)^3 (0.5)^2 = 10\,(0.5)^5$

$= 0.3125$//

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They expected number of family with 3 boys is $800 \times 0.3125 = 250$

(ii) $P(x=5) = {}^5C_5 (0.5)^5 (0.5)^{5-5}$

$= (1)(0.5)^5 (0.5)^0$

$= (1)(0.03125)(1)$

$= 0.03125$

They expected no. of family with 5 girls is $800 \times 0.03125 = 25$

(iii) $P(x=2) + P(x=3) = {}^5C_2 (0.5)^2 (0.5)^{5-2}$

$+ {}^5C_3 (0.5)^3 (0.5)^{5-3}$

$= (10)(0.5)^2 (0.5)^3 + 10 (0.5)^3 (0.5)^2$

$= (10)(0.5)^5 + 10 (0.5)^5$

$= 0.625$

They expected no. of family with 2 or 3 boys is 500

(iv) Atmost 2 girls means

$(P: x \leq 2)$ → family can have 5 boys & 0 girls
or 4 boys & 1 girl.
3 boys & 2 girl

$P(x=5) + P(x=4) + P(x=3)$

no. of boys

$= 0.03125 + {}^5C_4 (0.5)^4 (0.5) + 0.3125$

$= 0.5$

Expected no. of families with atmost 2 girls is $800 \times 0.5 = 400$

(10) Four coins are tossed 100 times and the following results were obtained fit a binomial distribution for the data & calculate the theoretical frequencies

No. of heads	0	1	2	3	4
Frequency	5	29	36	25	5

Sol^n: let x denote the number of heads and f the corresponding frequency since the data is in the form of a frequency distribution we shall first calculate the mean.

$$\text{mean}(\mu) = \frac{\sum fx}{\sum f} = \frac{0+29+72+75+20}{100}$$

$$= \frac{196}{100}$$

$$\mu = 1.96$$

$\mu = np$ for binomial distribution

$n=4$, $4p = 1.96 \Rightarrow p = 0.49$

four coins

w.k.T $p+q=1$

$q = 1-p$

$q = 1-0.49$

$q = 0.51$

we have $p(x) = {}^nC_x\, p^x q^{n-x}$

$p(x) = {}^4C_x\, (0.49)^x (0.51)^{4-x}$

Since 4 coins were tossed 100 times expected (theoretical) frequency are obtained from $F(x) = 100\,P(x)$

$= 100\; {}^4C_x\,(0.49)^x\,(0.51)^{4-x}$

where $x = 0, 1, 2, 3, 4$

$F(0) = 100\;{}^4C_0\,(0.49)^0\,(0.51)^{4-0} = 6.765 \approx 7$

$F(1) = 100\;{}^4C_1\,(0.49)^1\,(0.51)^{4-1} = 25.999 \approx 26$

$F(2) = 100\;{}^4C_2\,(0.49)^2\,(0.51)^{4-2} = 37.47 \approx 37$

$F(3) = 100\;{}^4C_3\,(0.49)^3\,(0.51)^{4-3} = 24.0004 \approx 24$

$F(4) = 100\;{}^4C_4\,(0.49)^4\,(0.51)^0 = 5.765 \approx 6$

$\therefore$ required theoretical frequency are 7, 26, 37, 24, 6

(11) The probability of shooter hitting a target is 1/3. How many times he should shoot so that the probability of hitting the target atleast once is morethan 3/4.

Soln: let p = Probability of hitting a target $= 1/3$

w.k.t $p + q = 1$

$q = 1 - p = 1 - 1/3 = 2/3$

$P(x) = {}^nC_x\,p^x q^{n-x}$

$P(x) = {}^nC_x\,(1/3)^x\,(2/3)^{n-x}$

we have to find n $\ni$;

$P(x \geq 1) > 3/4$

$1 - P(x < 1) > 3/4$

$1 - P(x=0) > 3/4$

$1 - ({}^nC_0\,p^0 q^n) > 3/4$

$1 - (q)^n > 3/4$

$1 - (2/3)^n > 3/4$

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$$-(+2/3)^n > \frac{3}{4} - 1$$

$$-(2/3)^n > -\frac{1}{4}$$

$$(2/3)^n < 1/4$$

$0.67 \nless 0.25$

$2/3 = 0.67, \quad 1/4 = 0.25$

we can find n by inspection method

$(2/3)^1 = 0.67, \quad (2/3)^2 = 0.44, \quad (2/3)^3 = 0.3,$

$(2/3)^4 = 0.2$

now $0.2 < 0.25$

$\therefore \underline{\underline{n = 4}}$

Poisson Distribution:-

It is regarded as the limiting form of the binomial distribution when n is very large ($n \geq 20$) & p the probability of success is very small ($q \to 0$) so that np tends to fixed finite constant say m.

Then $P(x) = \frac{e^{-m} m^x}{x!}$, where $m = np$

To show that P.D is a probability f^n

it satisfies the conditions ① $P(x) \geq 0$ ② $\sum P(x) = 1$

Now $\sum P(x) = \sum \frac{e^{-m} m^x}{x!}$

let us take $x = 0, 1, 2, 3 \ldots\ldots$

x	0	1	2	3	......
$P(x)$	e^{-m}	$\frac{e^{-m} m}{1!}$	$\frac{e^{-m} m^2}{2!}$	$\frac{e^{-m} m^3}{3!}$	+ ......

$= e^{-m}\left(1 + \frac{m}{1!} + \frac{m^2}{2!} + \frac{m^3}{3!} + \ldots\right)$

we have $p(x) \geq 0$ and

$$\sum p(x) = e^{-m} + \frac{me^{-m}}{1!} + \frac{m^2 e^{-m}}{2!} + \frac{m^3 e^{-m}}{3!} + \cdots$$

$$= e^{-m}\left\{1 + \frac{m}{1!} + \frac{m^2}{2!} + \frac{m^3}{3!} + \cdots\right\}$$

$$= e^{-m} \cdot e^{m}$$

$$= e^{-m+m} = e^{0}$$

$\sum p(x) = 1$, both the conditions ① $p(x) \geq 0$ ② $\sum p(x) = 1$ are satisfied, hence $p(x)$ is a probability f^n

Mean & Standard deviation of poisson distribution

$$\text{Mean}(\mu) = \sum_{x=0}^{\infty} x\,p(x)$$

$$= \sum_{x=0}^{\infty} [x] \frac{m^x e^{-m}}{x!}$$

$$= \sum_{x=0}^{\infty} \frac{\cancel{x} m^x e^{-m}}{\cancel{x}(x-1)!}$$

$$= \sum_{x=1}^{\infty} \frac{m^{x-1+1}}{(x-1)!} e^{-m}$$

$$= \sum_{x=1}^{\infty} \frac{m^{x-1} . m . e^{-m}}{(x-1)!}$$

$$= me^{-m} \sum_{x=1}^{\infty} \frac{m^{x-1}}{(x-1)!}$$

$$= me^{-m}\left[1 + \frac{m}{1!} + \frac{m^2}{2!} + \frac{m^3}{3!} + \dots\right]$$

$$= me^{-m}e^{m}$$

$$= me^{-m+m}$$

$$= me^{0}$$

$$= m$$

$\therefore$ mean $(\mu) = m$ //

$$\text{Variance}(V) = \sum_{x=0}^{\infty} x^2 p(x) - \mu^2 \quad \text{—} ①$$

Now $$\sum_{x=0}^{\infty} x^2 p(x) = \sum_{x=0}^{\infty} [x(x-1) + x] p(x)$$

$$= \sum_{x=0}^{\infty} [x(x-1)p(x) + x p(x)]$$

$$= \sum_{x=0}^{\infty} x(x-1)p(x) + \sum_{x=0}^{\infty} x p(x)$$

$$= \sum_{x=0}^{\infty} x(x-1)p(x) + m$$

$$= \sum_{x=0}^{\infty} x(x-1)\frac{m^x e^{-m}}{x!} + m$$

$$= \sum_{x=0}^{\infty} \frac{\cancel{x(x-1)} m^x e^{-m}}{\cancel{x(x-1)}(x-2)!} + m$$

$$= \sum_{x=0}^{\infty} \frac{m^x e^{-m}}{(x-2)!} + m$$

$$= \sum_{x=2}^{\infty} \frac{m^{x-2+2} e^{-m}}{(x-2)!} + m$$

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$$= \sum_{x=2}^{\infty} \frac{m^{x-2} \cdot m^2 \cdot e^{-m}}{(x-2)!} + m$$

$$= m^2 e^{-m} \sum_{x=2}^{\infty} \frac{m^{x-2}}{(x-2)!} + m$$

$$= m^2 e^{-m} \left[1 + \frac{m}{1!} + \frac{m^2}{2!} + \frac{m^3}{3!} + \dots \right] + m$$

$$= m^2 e^{-m} e^{m} + m$$

$$= m^2 e^{-m+m} + m$$

$$= m^2 e^{0} + m$$

Variance $= m^2 + m$

$$\therefore \sum x^2 p(x) = m^2 + m$$

Now (1) become

Variance $V = m^2 + m - \mu^2$

$\boxed{\text{w.k.T } \mu = m}$

$$= \cancel{m^2} + m - \cancel{m^2}$$

variance, $V = m$

$$S.D(\sigma) = \sqrt{V} = \sqrt{m}$$

hence mean $\mu = m$, Variance$(V) = m$,
& $S.D(\sigma) = \sqrt{m}$

we can say that mean & variance are equals for the poisson distribution

Problems

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(1) 2% of the fuses manufactured by a firm are found to be defective find the probability that a box containing 200 fuses contains
(i) no defective fuses
(ii) 3 or more defective fuses.

Sol: p = probability of defective fuse
$= 2/100$
$= 0.02$

$\therefore$ mean no. of defectives $\mu = m = np$
$= 200 \times 0.02$
$\mu = m = 4$
$\therefore m = 4$

poisson distribution is given by

$$P(x) = \frac{m^x e^{-m}}{x!}$$

$$= \frac{4^x e^{-4}}{x!}$$

$$p(x) = \frac{4^x (0.0183)}{x!}$$

(i) prob (no defective fuses) $= P(x=0)$ or $P(0)$

$$= \frac{4^0 (0.0183)}{0!}$$

$$= \frac{(1)\,0.0183}{(1)}$$

$$= 0.0183$$

(ii) prob. (3 or more defective fuses)

$= P(x \geq 3)$
$= 1 - P(x \leq 3)$

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$= 1 - [p(0) + p(1) + p(2)]$

$= 1 - \left[0.0183 \cdot \frac{4^0}{0!} + 0.0183 \cdot \frac{4^1}{1!} + 0.0183 \cdot \frac{4^2}{2!}\right]$

$= 1 - 0.0183 \left[\frac{(1)}{(1)} + \frac{4}{1} + \frac{16}{2}\right]$

$= 1 - 0.0183 [1 + 4 + 8]$

$= 1 - 0.2379$

$= 0.7621$

② Fit a poisson distribution for the following data & calculate the theoretical frequencies.

x	0	1	2	3	4
f	122	60	15	2	1

Solⁿ: $p(x) = \frac{e^{-m} m^x}{x!}$

$\mu = m = \frac{\sum fx}{\sum f} = \frac{0(122) + (1)(60) + 2(15) + 3(2) + 4(1)}{122 + 60 + 15 + 2 + 1}$

$= \frac{60 + 30 + 6 + 4}{200}$

$\mu = m = \frac{100}{200}$

$\boxed{m = 0.5}$

Now $p(x) = \frac{e^{-0.5} (0.5)^x}{x!}$

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Now we have to find theoretical frequency

Let $f(x) = P(x)\Sigma f$

$$f(x) = P(x)\,200 = \frac{e^{-0.5}(0.5)^{x}}{x!}\cdot 200$$

put $x = 0, 1, 2, 3, 4$

$$f(0) = 200\,\frac{e^{-0.5}(0.5)^{0}}{0!} = 200 \times 0.6065 = 121.3$$
$$\approx 121$$

$$f(1) = 200\,\frac{e^{-0.5}(0.5)^{1}}{1!} = 200 \times 0.6065 \times 0.5$$

$$f(1) = 60.65 \approx 61$$

$$f(2) = 200\,\frac{e^{-0.5}(0.5)^{2}}{2!} = \frac{200 \times 0.6065 \times 0.25}{2} = 15.1625$$

$$f(3) = 200\,\frac{e^{-0.5}(0.5)^{3}}{3!} = \frac{200 \times 0.6065 \times 0.125}{6}$$

$$f(3) = 2.527 \approx 3$$

$$f(4) = 200\,\frac{e^{-0.5}(0.5)^{4}}{4!} = \frac{200 \times 0.6065 \times 0.0625}{24}$$

$$f(4) = 0.3157 \approx 0$$

$\therefore$ Theoretical frequency are 121, 61, 15, 3, 0

③ A no. of accidents in a year to taxi drivers in a city follows a poisson distribution with mean 3. out of 1000 taxi drivers find approximetly the number of the drivers with ① no accident in a year ② morethan 3 accident in a year

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By data mean $(\mu) = 3$ i.e $\mu = m = 3$

$\boxed{m = 3}$

poisson distribution is given by

$$p(x) = \frac{m^x e^{-m}}{x!}$$

$$p(x) = \frac{3^x e^{-3}}{x!}$$

(i) no accident in a year

$$P(x=0) = \frac{3^0 e^{-3}}{0!} = \frac{(1)(0.04978)}{1} = 0.04978$$

Number of drivers out of 1000 with no accident in a year is 1000×0.04978

$= 49.78$

$\simeq 50$

(ii) morethan 3 accidents in a year

$P(x > 3) = 1 - P(x \leq 3)$

$= 1 - [P(0) + P(1) + P(2) + P(3)]$

$$= 1 - \left[\frac{3^0 e^{-3}}{0!} + \frac{3 e^{-3}}{1!} + \frac{3^2 e^{-3}}{2!} + \frac{3^3 e^{-3}}{3!}\right]$$

$$= 1 - e^{-3}\left[1 + 3 + \frac{9}{2} + \frac{27}{6}\right]$$

$= 1 - (0.04978)(13)$

$= 1 - (0.64714)$

$= 0.3528$

No. of drivers out of 1000 with morethan 3 accidents in a year

$= 1000 \times 0.3528$

$= 352.86 \simeq 353$

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Do yourself

(4) The number of accident per day (x) as recorded in a textile industry over a period of 400 days is given fit a poisson distribution for the data & Calculate the theoretical frequencies

x	0	1	2	3	4	5
f	173	168	37	18	3	1

Solⁿ Refer problem no. (2)

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(5) A shop has 4 disel generator sets which its hirey every day. The demand for a generator set on an average is a poisson variate with value 5/2. Obtain the probability that on a particular day (1) there was no demand (2) a demand had to be refused

Sol^n: By data mean $\mu = m = 5/2 = 2.5$

$\therefore m = 2.5$

Poisson distribution is given by

$$P(x) = \frac{m^x e^{-m}}{x!}$$

$$P(x) = \frac{(2.5)^x e^{-2.5}}{x!}$$

(i) No demand for generator

$$P(x=0) = P(0) = \frac{(2.5)^0 e^{-2.5}}{0!} = 0.082085$$

(ii) If a demand had to be refused, there should have been a demand for morethan 4 generators.

we have to find $P(x>4)$

$$P(x>4) = 1 - P(x \leq 4)$$

$$= 1 - \left[P(0) + P(1) + P(2) + P(3) + P(4)\right]$$

$$= 1 - \left[\frac{(2.5)^0 e^{-2.5}}{0!} + \frac{(2.5)^1 (e^{-2.5})}{1!} + \frac{(2.5)^2 e^{-2.5}}{2!} + \frac{(2.5)^3 e^{-2.5}}{3!} + \frac{(2.5)^4 e^{-2.5}}{4!}\right]$$

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$$= 1 - e^{-2.5}\left[1 + 2.5 + \frac{(2.5)^2}{2!} + \frac{(2.5)^3}{3!} + \frac{(2.5)^4}{4!}\right]$$

$p(x>4) = 0.10882$

(6) The probability that a news reader commits no mistake in reading the news is $1/e^3$. Find the probability that on a particular news broadcast he commits (1) only 2 mistakes
(2) more than 3 mistakes
(3) atmost 3 mistakes

Soln: By data
probability that a news reader commits no mistake $P(x=0)$ or $P(0) = 1/e^3 = e^{-3}$

Now poisson distribution is

$$P(x) = \frac{m^x e^{-m}}{x!}$$

where x denotes commiting mistake

we have $e^{-m} = e^{-3} \Rightarrow -m = -3 \Rightarrow m = 3$

$$\therefore P(x) = \frac{3^x e^{-3}}{x!}$$

(1) probability of commiting 2 mistakes

$$P(x=2) \text{ or } P(2) = \frac{3^2 e^{-3}}{2!} = 0.0498 \times \frac{9}{2}$$

$P(2) = 0.2241$

(2) probability of committing morethan 3 mistakes is $P(x>3)$

$$P(x>3) = 1 - P(x \leq 3)$$

$$= 1 - [P(0) + P(1) + P(2) + P(3)]$$

$= 1 - e^{-3}\left[1 + 3 + 9/2 + 27/6\right]$

$= 1 - e^{-3}(13)$

$p(x>3) = 0.3528$

(3) probability of commiting atmost 3 mistakes

$P(x \le 3) = P(0) + P(1) + P(2) + P(3)$

$$= e^{-3} + \frac{e^{-3}\,3^1}{1!} + \frac{e^{-3}\,3^2}{2!} + \frac{e^{-3}\,3^3}{3!}$$

$$= e^{-3}\left[1 + 3 + \frac{9}{2} + \frac{27}{6}\right]$$

$p(x \le 3) = 0.6472$

(7) If the probability of a bad reaction from a Certain injection is 0.001, determine the chance that out of 2000 individuals more than two will get a bad reaction.

Soln: poisson distribution is given by

$$p(x) = \frac{m^x e^{-m}}{x!}$$

$n = 2000$

$p = 0.001$

mean $\mu = m = np = 2000 \times 0.001$

$m = 2$

we have to find $P(x>2)$

$P(x>2) = 1 - P(x \le 2)$

$= 1 - [P(0) + P(1) + P(2)]$

w.K.T $p(x) = \frac{2^x e^{-2}}{x!}$

$= 1 - \left[\frac{e^{-2} 2^0}{0!} + \frac{e^{-2} 2^1}{1!} + \frac{e^{-2} 2^2}{2!} \right]$

$= 1 - e^{-2} \left[1 + 2 + 4/2 \right]$

$= 1 - e^{-2} \left[1 + 2 + 2 \right]$

$p(x>2) = 0.3233$

(8) Do yourself,

In a certain factory turning out razor blades there is a small probability of 1/500 for any blade to be defective the blades are supplied in packets of 10. Use poisson distribution to calculate the approximate no. of packets containing (i) no defective

(ii) one defective

(iii) two defective blades in a consignment of 10,000 packets.

Soln: probability of a defective blade

$p = 1/500 = 0.002$

$n = 10$

Mean $\mu = m = np = 10 \times 0.002 = 0.02$

$m = 0.02$

$p(x) = \frac{m^x e^{-m}}{x!}$

$p(x) = \frac{(0.02)^x e^{-0.02}}{x!}$ | $e^{-0.02} = 0.9802$

$p(x) = \frac{(0.02)^x (0.9802)}{x!}$

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(i) prob. of no defective $= P(x=0) = \frac{0.9802 \times (0.02)^0}{0!}$

$p(x=0)$ or $p(0) = 0.9802$

No/. of packets containing no defective blades out of 10,000

$= 0.9802 \times 10,000$

$= 9,802$

(ii) probability of one defective $= P(x=1)$

(or) $P(1) = \frac{0.9802 \times (0.02)^1}{1!} = 0.0196$

No/. of packets containing one defective blade out of $10,000 = 0.0196 \times 10,000$

$= 196$

(iii) probability of two defective $= P(x=2)$ or $P(2)$

$= \frac{0.9802 \times (0.02)^2}{2!}$

$P(2) = 0.00019$

No/. of packets containing two defective blades out of $10,000 = 10,000 \times 0.00019$

$= 1.9$

$\simeq 2$

OR

$p = 1/500 = 0.002$

mean $m = np = 10 \times 0.002 = 0.02$

you can solve like this

$P(x) = \frac{m^x e^{-m}}{x!} = \frac{(0.02)^x e^{-0.02}}{x!}$

Let $f(x) = 10,000\, P(x)$

$f(x) = 10,000 \frac{(0.02)^x e^{-0.02}}{x!}$

(i) probability of no defective $f(0) =$

$= 10,000 \frac{(0.02)^0 e^{-0.02}}{0!} = 9802$

(ii) probability of one defective $= f(1)$.

$= 10,000 \times \frac{(0.02) e^{-0.02}}{1!}$

$= 196$

(iii) probability of two defectives

$f(2) = 10,000 \frac{(0.02)^2 e^{-0.02}}{2!} = 1.96 \approx 2$

(9) If X follows a poisson variate $\ni$: $p(X=2) = 2/3\, p(X=1)$, find $p(x=0)$ and $p(x=3)$

Soln: we have $p(X=x) = \frac{m^x e^{-m}}{x!}$

By data

$P(X=2) = 2/3\, P(X=1)$

$\frac{m^2 e^{-m}}{2!} = \frac{2}{3} \frac{m e^{-m}}{1!}$

$\frac{m}{2} = \frac{2}{3} \Rightarrow m = 4/3$

hence $p(x=x) = \frac{m^x e^{-m}}{x!}$, $m = 4/3$

$P(X=x) = \frac{(4/3)^x e^{-4/3}}{x!}$

$P(X=0) = \frac{(4/3)^0 e^{-4/3}}{0!} = e^{-4/3} = 0.2636$

$P(X=3) = \frac{(4/3)^3 e^{-4/3}}{3!} = 0.10414$

(10) If x is a poisson variate $\ni$; $P(X=2) = 9P(X=4) + 90P(X=6)$, compute the mean & variance of poisson distribution.

Sol: we have $p(x) = \frac{m^x e^{-m}}{x!}$

by using the data we have

$P(X=2) = 9P(X=4) + 90P(X=6)$

$$\frac{m^2 e^{-m}}{2!} = 9\,\frac{m^4 e^{-m}}{4!} + 90\,\frac{m^6 e^{-m}}{6!}$$

$$\frac{\cancel{m^2}\cancel{e^{-m}}}{2} = \cancel{m^2}\cancel{e^{-m}}\left[\frac{9m^2}{24} + \frac{90\,m^4}{720}\right]$$

$$\frac{1}{2} = \frac{\overset{3}{\cancel{9}}\,m^2}{\underset{8}{\cancel{24}}} + \frac{m^4}{8}$$

$$\frac{1}{2} = \frac{3}{8}m^2 + \frac{m^4}{8}$$

$$1 = \frac{3}{4}m^2 + \frac{m^4}{4}$$

$$4 = 3m^2 + m^4$$

$$m^4 + 3m^2 - 4 = 0$$

$-4m^4$ → $+4m^2$, m^2

$\cancel{(m^2)^2 + (2)^2}$.

$$m^4 + 4m^2 - m^2 - 4 = 0$$

$$m^2(m^2+4) - 1(m^2+4) = 0$$

$$(m^2+4)(m^2-1) = 0$$

$m^2 + 4 = 0$ or $m^2 - 1 = 0$

$m^2 = -4$, $m^2 = 1$

$m = \pm 2i$, $m = \pm 1$

Neglect complex root (value) & negative values.

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$\therefore m = 1$

variance and mean are equal in poisson distribution

$\therefore m = \text{variance} = 1$

variance and mean are equal in poisson distribution

$\therefore m = \text{variance} = 1$

Contineous probability distribution

If every x belonging to the range of contineous random variable 'x' we assign a real no, $f(x)$ satisfying the conditions ① $f(x) \geq 0$

② $\int_{-\infty}^{\infty} f(x)\, dx = 1$

then $f(x)$ is called a contineous probability function or probability density function

Cumulative distribution function:-

If X is a contineous random variable with probability density function $f(x)$ then $F(x)$ is defined by

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)\, dx \quad \text{is}$$

called cumulative distribution function

NOTE:

$$\frac{d}{dx}\left[F(x)\right] = f(x)$$

$$P(x \geq r) = \int_{r}^{\infty} f(x)\, dx$$

$$P(x < r) = 1 - P(x \geq r)$$

$$P(x < r) = 1 - \int_{r}^{\infty} f(x)\, dx$$

Mean & Variance of Contineous Probability distribution

$$\text{mean } (\mu) = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$\text{Variance } (\sigma^2) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx$$

Exponential distribution:-

The contineous probability distribution having the probability density function $f(x)$ given by

$$f(x) = \begin{cases} \alpha e^{-\alpha x} & \text{for } x > 0 \\ 0 & \text{otherwise, where } \alpha > 0 \end{cases}$$

is known as exponential distribution

If $f(x) > 0$ & we have

$$\int_{-\infty}^{\infty} f(x)\, dx = \int_0^{\infty} \alpha e^{-\alpha x}\, dx$$

$$= \alpha \int_0^{\infty} e^{-\alpha x}\, dx$$

$$= \alpha \left[\frac{e^{-\alpha x}}{-\alpha} \right]_0^{\infty}$$

$$= \frac{\alpha}{-\alpha} \left[e^{-\alpha x} \right]_0^{\infty}$$

$$= -\left[e^{-\infty} - e^{0} \right]$$

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$= -[0-1]$

$= 1$

$$\therefore \boxed{\int_{-\infty}^{\infty} f(x)\,dx = 1}$$

f(x) satisfy both the conditions required for a Contineous probability function / probability density function.

mean & Variance of Exponential distribution

$$\text{mean}(\mu) = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \int_{0}^{\infty} x \alpha e^{-\alpha x}\,dx$$

$$= \alpha \int_{0}^{\infty} x e^{-\alpha x}\,dx$$

Applying Bernoulli's rule of integration by parts we have

$$\mu = \alpha \left[x\left(\frac{e^{-\alpha x}}{-\alpha}\right) - (1)\left(\frac{e^{-\alpha x}}{\alpha^2}\right) \right]_0^{\infty}$$

$$\mu = \alpha \left[(0-0) - \left(0 - \left(+\frac{1}{\alpha^2}\right)\right) \right]$$

$$= \alpha \left[0 + \frac{1}{\alpha^2} \right]$$

$$\mu = \alpha\left(\frac{1}{\alpha^2}\right)$$

$$\boxed{\mu = \frac{1}{\alpha}}$$

Variance $(\sigma^2) = \int_{-\infty}^{\infty} (x-\mu)^2 \cdot f(x)\, dx$

$$\sigma^2 = \int_0^{\infty} (x-\mu)^2 \alpha e^{-\alpha x}\, dx$$

$$\sigma^2 = \alpha \int_0^{\infty} (x-\mu)^2 e^{-\alpha x}\, dx$$

Applying Bernoulli's rule we have

$$\sigma^2 = \alpha\left[(x-\mu)^2 \frac{e^{-\alpha x}}{-\alpha} - 2(x-\mu)\frac{e^{-\alpha x}}{\alpha^2} + 2\frac{e^{-\alpha x}}{-\alpha^3}\right]_0^{\infty}$$

$$= \alpha\left[\frac{-1}{\alpha}\left(0-\mu^2\right) - \frac{2}{\alpha^2}\left(0-(-\mu)\right) - \frac{2}{\alpha^3}\left(0-1\right)\right]$$

$$= \alpha\left[\frac{\mu^2}{\alpha} - \frac{2\mu}{\alpha^2} + \frac{2}{\alpha^3}\right]$$

But $\mu = \frac{1}{\alpha}$

$$= \alpha\left[\frac{1/\alpha^2}{\alpha} - \frac{2 \cdot 1/\alpha}{\alpha^2} + \frac{2}{\alpha^3}\right]$$

$$= \alpha\left[\frac{1}{\alpha^3} \cancel{- \frac{2}{\alpha^3}} + \cancel{\frac{2}{\alpha^3}}\right]$$

$$\sigma^2 = \frac{1}{\alpha^2}$$

$\sigma = \frac{1}{\alpha}$

Thus for exponential distribution

mean $(\mu) = \frac{1}{\alpha}$; S.D $(\sigma) = \frac{1}{\alpha}$, variance $\sigma^2 = 1/\alpha^2$

NOTE: mean = S.D for the exponential distribution.

Problems

① Find which of the following functions is a probability density function.

(i) $f_1(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$

(ii) $f_2(x) = \begin{cases} 2x, & -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$

(iii) $f_3(x) = \begin{cases} |x|, & |x| \leq 1 \\ 0, & \text{otherwise} \end{cases}$

(iv) $f_4(x) = \begin{cases} 2x & 0 < x \leq 1 \\ 4-4x & 1 < x < 2 \\ 0 & \text{otherwise} \end{cases}$

Sol: To show that given functions are p.d.f it should satisfy the conditions

① $f(x) \geq 0$

② $\int_{-\infty}^{\infty} f(x)\,dx = 1$

(i) clearly $f(x) \geq 0$

$\int_{-\infty}^{\infty} f_1(x)\,dx = \int_0^1 f_1(x)\,dx = \int_0^1 2x\,dx = \left[\frac{\cancel{2}x^2}{\cancel{2}}\right]_0^1$

$= [x^2]_0^1$

$= [1-0]$

$\int_{-\infty}^{\infty} f_1(x)dx = 1$

$\therefore f_1(x)$ is a p.d.f

(ii) The given function can be written in the form

$$f_2(x) = \begin{cases} 2x, & -1 < x < 0 \\ 2x, & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

In $-1 < x < 0$, $f_2(x) = 2x$ is lessthan zero

$$\int_{-\infty}^{\infty} f_2(x)\,dx = \int_{-1}^{1} f_2(x)\,dx$$

$$= \int_{-1}^{1} 2x\,dx$$

$$= 2\left[\frac{x^2}{2}\right]_{-1}^{1}$$

$$= [1-(-1)^2]$$

$$= 1-1$$

$$= 0$$

both the Conditions are not Satisfied.

$\therefore f_2(x)$ is not a p.d.f.

(iii) Evidently $f_3(x) = |x| \geq 0$

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$$\int_{-\infty}^{\infty} f_3(x)\,dx = \int_{-1}^{1} f_3(x)\,dx = \int_{-1}^{1} |x|\,dx$$

$$|x| = \begin{cases} -x & -1 < x < 0 \\ x & 0 < x < 1 \end{cases}$$

$$\int_{-\infty}^{\infty} f_3(x)\,dx = \int_{-1}^{0} -x\,dx + \int_{0}^{1} x\,dx$$

$$= \left[-\frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_{0}^{1}$$

$$= -\tfrac{1}{2}\left[0-(-1)^2\right] + \tfrac{1}{2}\left[1-0\right]$$

$$= -\tfrac{1}{2}\left[-1\right] + \tfrac{1}{2}\left[1\right]$$

$$= \tfrac{1}{2} + \tfrac{1}{2}$$

$$= 1$$

$\therefore f_3(x)$ is p.d.f

(iv) $f_4(x) = 2x > 0$ in $0 < x \leq 1$

$f_4(x) = 4 - 4x$ is negative in $1 < x < 2$

The first condition is not satisfied

$f_4(x)$ is not a p.d.f

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(2) find the value of c such that

$$f(x) = \begin{cases} \frac{x}{6} + c, & 0 \le x \le 3 \\ 0 & , \text{otherwise} \end{cases}$$

is a probability density function
also find $P(1 \le x \le 2)$

p.d.f is valid

Solⁿ: $f(x) \geq 0$ if $c \geq 0$

also we must have $\int_{-\infty}^{\infty} f(x)dx = 1$

$$\int_0^3 \left(\frac{x}{6} + c\right) dx = 1$$

$$\left[\frac{x^2}{6 \times 2} + cx\right]_0^3 = 1$$

$$\left(\frac{3^2}{12} + c \cdot 3\right) - 0 = 1$$

$$\frac{\cancel{9}^{3}}{\cancel{12}_{4}} + 3c = 1$$

$$\frac{3}{4} + 3c = 1$$

$$3c = 1 - \frac{3}{4} = \frac{4-3}{4} = \frac{1}{4}$$

$$3c = \frac{1}{4} \Rightarrow c = \frac{1}{12}$$

Now $P(1 \le x \le 2) = \int_1^2 f(x)\, dx$

$$= \int_1^2 \left(\frac{x}{6} + c\right) dx$$

$$= \int_1^2 \left(\frac{x}{6} + \frac{1}{12}\right) dx$$

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$$= \left[\frac{x^2}{12} + \frac{1}{12}\cdot x\right]_1^2$$

$$= \left(\frac{4}{12} + \frac{2}{12}\right) - \left(\frac{1}{12} + \frac{1}{12}\right) \quad \text{OR} = \frac{1}{12}\left[x^2+x\right]_1^2$$

$$= \left(\frac{1}{3} + \frac{1}{6}\right) - \left(\frac{2}{12}\right) \qquad = \frac{1}{12}\left[(2^2+2)-(1+1)\right]$$

$$= \frac{2+1}{6} - \frac{2}{12} \qquad = \frac{1}{12}\left[6-2\right]$$

$$= \frac{3}{6} - \frac{1}{6} \qquad = \frac{1}{12} \times 4$$

$$= \frac{2}{6} \qquad P(1 \le x \le 2) = 1/3$$

$$P(1 \le x \le 2) = \frac{1}{3}$$

③ Find the Constant K Such that

$$f(x) = \begin{cases} Kx^2 ; & 0 < x < 3 \\ 0 , & \text{otherwise} \end{cases}$$ is a probabality density function also compute

① $P(1 < x < 2)$ ② $P(x \le 1)$ ③ $P(x > 1)$

④ mean ⑤ variance

Solⁿ: probability density function is valid $f(x) \ge 0$ if $K \ge 0$.

also we must have $\int_{-\infty}^{\infty} f(x)\, dx = 1$

$$\int_0^3 Kx^2\, dx = 1$$

$$K\int_0^3 x^2\, dx = 1$$

$$K\left[x^3/3\right]_0^3 = 1$$

$K\left[\frac{3^3}{3} - 0\right] = 1$

$K\left[3^2\right] = 1$

$9K = 1$

$K = \frac{1}{9}$

(i) $p(1 < x < 2) = \int_1^2 f(x)\,dx$

$= \int_1^2 Kx^2\,dx = \int_1^2 \frac{1}{9}x^2\,dx$

$= \frac{1}{9}\left[\frac{x^3}{3}\right]_1^2$

$= \frac{1}{9}\left[\frac{2^3}{3} - \frac{1}{3}\right]$

$= \frac{1}{9}\left[\frac{8}{3} - \frac{1}{3}\right] = \frac{1}{9} \times \frac{7}{3}$

$= \frac{7}{27}$

(ii) $p(x \leq 1) = \int_0^1 f(x)\,dx = \int_0^1 \frac{x^2}{9}\,dx = \left[\frac{x^3}{27}\right]_0^1$

$= \left[\frac{1}{27} - 0\right]$

$P(x \leq 1) = \frac{1}{27}$

(iii) $P(x > 1) = \int_1^3 f(x)\,dx = \int_1^3 \frac{x^2}{9}\,dx = \left[\frac{x^3}{27}\right]_1^3$

$= \left[\frac{3^3}{27} - \frac{1}{27}\right]$

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$$= \left[\frac{27}{27} - \frac{1}{27}\right]$$

$$= \left[1 - 1/27\right]$$

$$p(x>1) = \frac{26}{27}$$

(iv) mean $= \mu = \int_{-\infty}^{\infty} x\, f(x)\, dx$

$$= \int_0^3 x \cdot \frac{x^2}{9}\, dx$$

$$= \int_0^3 \frac{x^3}{9}\, dx$$

$$= \frac{1}{9}\int_0^3 x^3\, dx = \frac{1}{9}\left[\frac{x^4}{4}\right]_0^3$$

$$= \frac{1}{36}\left[3^4 - 0\right]$$

$$= \frac{1}{36}(81)$$

$$\mu = \frac{9}{4}$$

(v) Variance $= V = \int_{-\infty}^{\infty} x^2 f(x)\, dx - (\mu)^2$

$$= \int_0^3 x^2 \cdot \frac{x^2}{9}\, dx - \left(\frac{9}{4}\right)^2$$

$$= \int_0^3 \frac{x^4}{9}\, dx - \frac{81}{16}$$

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$$= \left[\frac{x^5}{9\times 5}\right]_0^3 - \frac{81}{16}$$

$$= \left(\frac{3^5}{45} - 0\right) - \frac{81}{16}$$

$$= \frac{\cancel{243}^{81}}{\cancel{45}_{15}} - \frac{81}{16}$$

$$= \frac{81}{15} - \frac{81}{16} = \frac{81}{240} = \frac{27}{80}$$

$$V = \frac{27}{80}$$ //

(4) A random variable x has the following density function

$$p(x) = \begin{cases} Kx^2, & -3 \le x \le 3 \\ 0, & \text{elsewhere} \end{cases}$$

Evaluate K & find (1) $p(1 \le x \le 2)$
(2) $p(x \le 2)$
(3) $p(x > 1)$

Sol^n: we must have (1) $p(x) \geq 0$ if $K \geq 0$

(2) $\int_{-\infty}^{\infty} p(x)\,dx = 1$

i.e $\int_{-3}^{3} Kx^2\,dx = 1$

$$K \int_{-3}^{3} x^2\,dx = 1$$

$$K\left[\frac{x^3}{3}\right]_{-3}^{3} = 1$$

$\frac{K}{3}\left[3^3-(-3)^3\right]=1$

$\frac{K}{3}\left[27-(-27)\right]=1$

$\frac{K}{3}\left[27+27\right]=1$

$\frac{K}{\cancel{3}}\left[\overset{18}{\cancel{54}}\right]=1$

$18K=1$

$K=\frac{1}{18}$ //

① $P(1 \le x \le 2) = \int_1^2 Kx^2\,dx = \int_1^2 \frac{1}{18}x^2\,dx$

$= \frac{1}{18}\int_1^2 x^2\,dx$

$= \frac{1}{18}\left[\frac{x^3}{3}\right]_1^2$

$= \frac{1}{18}\left[\frac{2^3}{3}-\frac{1}{3}\right]$

$= \frac{1}{18}\left[\frac{8}{3}-\frac{1}{3}\right]$

$= \frac{1}{18}\times\frac{7}{3}$

$= \frac{7}{54}$ //

② $P(x \le 2) = \int_{-3}^2 Kx^2\,dx = \int_{-3}^2 \frac{1}{18}x^2\,dx = \frac{1}{18}\int_{-3}^2 x^2\,dx$

$= \frac{1}{18}\left[\frac{x^3}{3}\right]_{-3}^2$

$= \frac{1}{18}\left[\frac{2^3}{3}-\frac{(-3)^3}{3}\right]$

$$= \frac{1}{18}\left[\frac{8}{3} - \frac{(-27)}{3}\right]$$

$$= \frac{1}{18}\left[\frac{8}{3} + \frac{27}{3}\right]$$

$$= \frac{1}{18} \times \frac{35}{3}$$

$$= \frac{35}{54}$$

③ $P(x>1) = \int_1^3 Kx^2\,dx = \int_1^3 \frac{1}{18}x^2\,dx$

$$= \frac{1}{18}\int_1^3 x^2\,dx$$

$$= \frac{1}{18}\left[\frac{x^3}{3}\right]_1^3$$

$$= \frac{1}{18}\left[\frac{3^3}{3} - \frac{1}{3}\right]$$

$$= \frac{1}{18}\left[\frac{27}{3} - \frac{1}{3}\right]$$

$$= \frac{1}{18} \times \frac{26}{3}$$

$$= \frac{26}{54}$$

$$P(x>1) = \frac{13}{27}$$

⑤ Find the CDF (cumulative distribution function) for probability density function of random variable X.

ⓘ $f(x) = \begin{cases} 6x - 6x^2, & 0 \le x \le 1 \\ 0 & ; \text{otherwise} \end{cases}$

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(ii) $f(x) = \begin{cases} \frac{x}{4} e^{-x/2}, & 0 < x < \infty \\ 0, & \text{otherwise} \end{cases}$

Soln: If $f(x)$ is the probability density function then c.d.f $= F(x) = \int_{-\infty}^{x} f(x)\,dx$

(i) $F(x) = \int_{-\infty}^{x} f(x)\,dx$

$= \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{x} f(x)\,dx$

$= 0 + \int_{0}^{x} f(x)\,dx$

$F(x) = \int_{0}^{x} (6x - 6x^2)\,dx$

$= \left[\frac{6x^2}{2} - \frac{6x^3}{3}\right]_0^x$

$= \left[3x^2 - 2x^3\right]_0^x$

$F(x) = 3x^2 - 2x^3$

$\therefore$ c.d.f $= 3x^2 - 2x^3$ if $0 \le x \le 1$ //

(ii) $F(x) = \int_{-\infty}^{x} f(x)\,dx$

$= \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{x} f(x)\,dx$

$= 0 + \int_{0}^{x} f(x)\,dx$

$$= \int_0^x \frac{x}{4} e^{-x/2}\, dx$$

Apply bernoulli's rule

$$= \frac{1}{4} \int_0^x x\, e^{-x/2}\, dx$$

$$= \frac{1}{4} \left[x \frac{e^{-x/2}}{-1/2} - (1) \frac{e^{-x/2}}{-\frac{1}{2} \times -\frac{1}{2}} \right]_0^x$$

$$= \frac{1}{4} \left[-2x e^{-x/2} - 4 e^{-x/2} \right]_0^x$$

$$= \frac{1}{4} \left[\left\{ -2x e^{-x/2} - 4 e^{-x/2} \right\} - \left\{ 0 - 4 e^{0} \right\} \right]$$

$$= \frac{1}{4} \left[-2 (x e^{-x/2} + 2 e^{-x/2}) + 4 \right]$$

$$= \frac{1}{4} \left[-2 e^{-x/2} (x + 2) + 4 \right]$$

$$= -\frac{1}{2} e^{-x/2} (x + 2) + 1$$

$$= -\frac{1}{2} e^{-x/2} . x + e^{-x/2} + 1$$

$$F(x) = 1 - \frac{x}{2} e^{-x/2} - e^{-x/2} \quad \text{if } 0 < x < \infty$$

⑥ A Contineous random variable has the distribution function

$$F(x) = \begin{cases} 0, & x \le 1 \\ c(x-1)^4, & 1 \le x \le 3 \\ 1, & x > 3 \end{cases}$$

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Find C and also the p.d.f

Soln: W.K.T p.d.f $f(x) = \frac{d}{dx}[F(x)]$

$$\therefore f(x) = \begin{cases} 0 ; & x \le 1 \\ 4C(x-1)^3 ; & 1 \le x \le 3 \\ 0 , & x > 3 \end{cases}$$

① $f(x) \geq 0$ for $C \geq 0$

② we must have $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\int_1^3 f(x)\,dx = 1$, $\int_1^3 4C(x-1)^3\,dx = 1$

$$4C \int_1^3 (x-1)^3\,dx = 1$$

$$4C \left[\frac{(x-1)^4}{4}\right]_1^3 = 1$$

$$C\left[(x-1)^4\right]_1^3 = 1$$

$$C\left[(3-1)^4 - (1-1)^4\right] = 1$$

$$C\left[2^4 - 0\right] = 1$$

$$C[16] = 1$$

$$C = 1/16$$

Thus p.d.f $f(x) = (x-1)^3 \cdot 4C = (x-1)^3 \cdot 4 \cdot \frac{1}{16}$

$= \frac{(x-1)^3}{4}$ where $1 \le x \le 3$

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(7) Find K so that the following function can serve as a p.d.f of a random variable $f(x) = \begin{cases} 0 & \text{for } x \le 0 \\ Kx e^{-4x^2} & \text{for } x > 0 \end{cases}$

Soln: we must have $f(x) \geq 0$ and

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \Rightarrow \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{\infty} f(x)\,dx = 1$$

$$\Rightarrow 0 + \int_{0}^{\infty} f(x)\,dx = 1$$

$$\therefore \int_{0}^{\infty} f(x)\,dx = 1 \Rightarrow \int_{0}^{\infty} Kx e^{-4x^2}\,dx = 1$$

$$\Rightarrow K \int_{0}^{\infty} x e^{-4x^2}\,dx = 1$$

put $4x^2 = t$

D.w.r. to x

$8x\,dx = dt$

$x\,dx = \frac{1}{8}\,dt$

when x varies from 0 to ∞

t also varies from 0 to ∞

$$\Rightarrow K \int_{0}^{\infty} e^{-t} \frac{dt}{8} = 1$$

$$\Rightarrow \frac{K}{8} \left[\frac{e^{-t}}{-1}\right]_{0}^{\infty} = 1$$

$$\Rightarrow \frac{-K}{8} \left[e^{-t}\right]_{0}^{\infty} = 1$$

$\Rightarrow -\frac{K}{8}\left[0-1\right]=1$

$\frac{K}{8}=1$

$\underline{K=8}$

(8) The kilometre run (in thousands of kms) without any sort of problem in respect of a certain vehicle is a random variable having p.d.f

$$f(x)=\begin{cases}\frac{1}{40}e^{-x/40}, & x\geq 0\\ 0, & x\leq 0\end{cases}$$

Find the probabality that vehicle is trouble free

(1) atleast for 25000 kms

(2) atmost for 25000 kms

(3) between 16000 to 32000 kms

Sol: Here x is the random variable representing kilometre in multiples of 1000 regarding trouble free run by the vehicle

(1) To find $P(x\geq 25)$

$P(x\geq 25)=1-P(x<25)$

$$P(x\geq 25)=1-\int_0^{25}\frac{1}{40}e^{-x/40}\,dx$$

$$=1-\cancel{\frac{1}{40}}\left[\frac{e^{-x/40}}{\cancel{-1/40}}\right]_0^{25}$$

$$=1+\left[e^{-x/40}\right]_0^{25}$$

$= 1 + [e^{-25/40} - e^{0}]$

$= 1 + [e^{-5/8} - 1]$

$= e^{-5/8}$

(ii) $P(x \leq 25)$

$P(x \leq 25) = \int_0^{25} \frac{1}{40} e^{-x/40} \, dx$

$= \frac{1}{40} \left[\frac{e^{-x/40}}{-1/40} \right]_0^{25}$

$= -[e^{-25/40} - e^{0}]$

$= -[e^{-5/8} - 1]$

$= -e^{-5/8} + 1$

$P(x \leq 25) = 0.4647$

(iii) $P(16 \leq x \leq 32) = \int_{16}^{32} \frac{1}{40} e^{-x/40} \, dx$

$= \frac{1}{40} \left[\frac{e^{-x/40}}{-1/40} \right]_{16}^{32}$

$= -[e^{-x/40}]_{16}^{32}$

$= -[e^{-32/40} - e^{-16/40}]$

$= -[e^{-4/5} - e^{-2/5}]$

$= 0.221$

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(9) If x is an exponential variate with mean 3 find (1) $p(x>1)$
(2) $p(x<3)$

Sol: p.d.f of the exponential distribution is given by $f(x) = \begin{cases} \alpha e^{-\alpha x}, & 0<x<\infty \\ 0, & \text{otherwise} \end{cases}$

mean of exponential distribution is

$$\mu = \frac{1}{\alpha}$$

given $\mu = 3$

$$\frac{1}{\alpha} = 3 \Rightarrow \alpha = 1/3$$

hence $f(x) = \begin{cases} \frac{1}{3} e^{-\frac{1}{3}x}, & 0<x<\infty \\ 0, & \text{otherwise} \end{cases}$

(1) $P(x>1) = 1 - P(x \le 1)$

$$= 1 - \int_0^1 f(x)\,dx$$

$$= 1 - \int_0^1 \frac{1}{3} e^{-\frac{1}{3}x}\,dx$$

$$= 1 - \frac{1}{3}\left(\frac{e^{-\frac{1}{3}x}}{-1/3}\right)_0^1$$

$$= 1 + \left[e^{-1/3} - e^0\right]$$

$$= \cancel{1} + e^{-1/3} - \cancel{1}$$

$$= e^{-1/3} = 0.7165$$

(2) $P(x<3) = \int_0^3 f(x)\,dx = \int_0^3 1/3\, e^{-x/3}\,dx$

$$= 1/3 \left[\frac{e^{-x/3}}{-1/3}\right]_0^3$$

$$= -\left[e^{-1} - e^0\right] = -\left[e^{-1} - 1\right] = 1 - e^{-1}$$

$p(x<3) = 0.6321$

1

(10) In a Certain town, the duration of a shower is exponentially distributed with mean 5 minutes. what is the probability that a shower will last for ① 10 minutes or more
② Less than 10 minutes
③ between 10 & 12 minutes

Sol^{n}: Given $\mu = 5$

$\mu = \frac{1}{\alpha}$, mean of exponential distribution

$\therefore \frac{1}{\alpha} = 5 \Rightarrow \alpha = \frac{1}{5} = 0.2$

probability density function of exponential distribution is given by

$$f(x) = \begin{cases} \alpha e^{-\alpha x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

$$f(x) = 0.2 e^{-0.2x}$$

we have to find

① $p(x \geq 10) = \int_{10}^{\infty} f(x)\,dx$

$$= \int_{10}^{\infty} 0.2 e^{-0.2x}\,dx$$

$$= \cancel{0.2}\left[\frac{e^{-0.2x}}{\cancel{-0.2}}\right]_{10}^{\infty}$$

$$= -\left[e^{-\infty} - e^{-0.2\times 10}\right]$$

$$p(x \geq 10) = -\left[0 - e^{-2}\right] = e^{-2} = \underline{0.1353}$$

(ii) $P(x<10) = \int_0^{10} f(x)\, dx$

$$= \int_0^{10} 0.2\, e^{-0.2x}\, dx$$

$$= \cancel{0.2} \left[\frac{e^{-0.2x}}{\cancel{-0.2}} \right]_0^{10}$$

$$= -\left[e^{-0.2\times10} - e^{0} \right]$$

$$= -\left[e^{-2} - 1 \right]$$

$$= 1 - e^{-2}$$

$$= 0.8647$$

(iii) $P(10<x<12) = \int_{10}^{12} f(x)\, dx$

$$= \int_{10}^{12} 0.2\, e^{-0.2x}\, dx$$

$$= \cancel{0.2} \left[\frac{e^{-0.2x}}{\cancel{-0.2}} \right]_{10}^{12}$$

$$= -\left[e^{-0.2\times12} - e^{-0.2\times10} \right]$$

$$= -\left[e^{-2.4} - e^{-2} \right]$$

$$= \underline{0.0446}$$

(11) The length of telephone conversation in a booth has been an exponential distribution and found on an average to be 5 minutes. find the probability that a random call made from this booth ① ends less than 5 minutes
② b/w 5 and 10 minutes

Sol: Exponential distribution of mean is $\mu = \frac{1}{\alpha}$

Given $\mu = 5$

$$\frac{1}{\alpha} = 5 \Rightarrow \alpha = \frac{1}{5} = 0.2$$

p.d.f of exponential distribution is given by $f(x) = \begin{cases} \alpha e^{-\alpha x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$

$$f(x) = \alpha e^{-\alpha x}$$
$$f(x) = 0.2 e^{-0.2x}$$

① $P(x < 5) = \int_0^5 f(x)\, dx$

$$= \int_0^5 0.2 e^{-0.2x}\, dx$$
$$= 0.2 \int_0^5 e^{-0.2x}\, dx$$
$$= \cancel{0.2} \left[\frac{e^{-0.2x}}{-\cancel{0.2}} \right]_0^5$$
$$= -\left[e^{-0.2\times5} - e^{-0.2\times0} \right]$$
$$= -\left[e^{-1} - e^{0} \right]$$

$= -[e^{-1} - 1]$

$= 0.6321$

② $P(5 < x < 10) = \int_5^{10} f(x)\,dx$

$= \int_5^{10} 0.2\, e^{-0.2x}\, dx$

$= 0.2 \int_5^{10} e^{-0.2x}\, dx$

$= \cancel{0.2} \left[\frac{e^{-0.2x}}{\cancel{-0.2}} \right]_5^{10}$

$= - \left[e^{-0.2\times10} - e^{-0.2\times5} \right]$

$= - \left[e^{-2} - e^{-1} \right]$

$= 0.2325$

Do yourself

If x is an exponential variate with mean 5, evaluate

① $P(0 < x < 1)$ Ans : 0.1813

② $P(-\infty < x < 10)$: 0.8647

③ $P(x \leq 0 \text{ or } x \geq 1)$: 0.8187

$\hookrightarrow P(x \leq 0) + P(x \geq 1)$

$\hookrightarrow 0 + \int_1^{\infty} f(x)\,dx$

(12) Find K so that $f(x) = \begin{cases} Kxe^{-x}, & 0<x<1 \\ 0, & \text{otherwise} \end{cases}$

is a p.d.f. Find the mean.

Sol^n: $f(x) \geq 0$ if $K \geq 0$

we must have $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_0^1 Kxe^{-x}\,dx = 1$$

$$K\int_0^1 xe^{-x}\,dx = 1$$

apply Bernoulli's rule

$$K\left[\frac{xe^{-x}}{-1} - e^{-x}\right]_0^1 = 1$$

$$K\left[\left\{\frac{e^{-1}}{-1} - e^{-1}\right\} - \left\{0 - e^{0}\right\}\right] = 1$$

$$K\left[-1/e - 1/e + 1\right] = 1$$

$$K\left[-2/e + 1\right] = 1$$

$$K\left[1 - 2/e\right] = 1$$

$$K = \frac{1}{1-2/e} = \frac{e}{e-2}$$

$$K = \frac{e}{e-2}$$

$$\mu = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$= \int_0^1 x \cdot \frac{e}{e-2}\, x e^{-x}\, dx$$

$$= \frac{e}{e-2} \int_0^1 x^2 e^{-x}\, dx$$

$$= \frac{e}{e-2} \left[x^2 \frac{e^{-x}}{-1} - 2x \frac{e^{-x}}{(-1)(-1)} + 2 \frac{e^{-x}}{(-1)(-1)(-1)} \right]_0^1$$

$$= \frac{e}{e-2} \left[\left\{ \frac{e^{-1}}{-1} - 2\frac{e^{-1}}{1} + 2\frac{e^{-1}}{-1} \right\} - \left\{ 0 - 0 - 2 \right\} \right]$$

$$= \frac{e}{e-2} \left[-e^{-1} - 4e^{-1} + 2 \right]$$

$$= \frac{e}{e-2} \left[2 - 5e^{-1} \right]$$

$$= \frac{e}{e-2} \left[2 - 5/e \right]$$

$$= \frac{\cancel{e}}{e-2} \left[\frac{2e-5}{\cancel{e}} \right]$$

$$= \frac{2e-5}{e-2}$$ //

②

(8) Is the following function a p.d.f?

$$f(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

If so determine the probability that the variate having this density will fall in the interval (1, 2).

Sol^n: we observe $f(x) \geq 0$.

also we must have $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{\infty} f(x)\,dx$$

$$= 0 + \int_{0}^{\infty} e^{-x}\,dx$$

$$= \left[-e^{-x}\right]_{0}^{\infty}$$

$$= -[0-1]$$

$$= 1$$

$\therefore f(x)$ is a p.d.f^n.

Normal Distribution

The Continuous probability distribution having probability density function $f(x)$ given by $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$

where $-\infty < x < \infty$, $-\infty < \mu < \infty$ and $\sigma > 0$ is known as normal distribution.

evidently $f(x) \geq 0$ we are showing it is a p.d.f

$$\int_{-\infty}^{\infty} f(x)\,dx = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-(x-\mu)^2/2\sigma^2}\,dx$$

put $t = \frac{x-\mu}{\sqrt{2}\sigma}$ or $x = \mu + \sqrt{2}\sigma t$,

we have $dx = \sqrt{2}\sigma\,dt$

t also varies from $-\infty$ to ∞

hence $\int_{-\infty}^{\infty} f(x)\,dx = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-t^2}\sqrt{2}\sigma\,dt$

$$= \frac{1}{\cancel{\sigma}\cancel{\sqrt{2}}\sqrt{\pi}} \cdot \cancel{\sqrt{2}} \int_{-\infty}^{\infty} e^{-t^2} \cdot \cancel{\sigma}\,dt$$

$$= \frac{1}{\sqrt{\pi}} \times 2 \int_{0}^{\infty} e^{-t^2}\,dt$$

But $\int_0^{\infty} e^{-t^2}\,dt = \frac{\sqrt{\pi}}{2}$ by gamma functions

$$\int_{-\infty}^{\infty} f(x)\,dx = \frac{\cancel{2}}{\cancel{\sqrt{\pi}}} \times \frac{\cancel{\sqrt{\pi}}}{\cancel{2}} = 1$$

both the conditions satisfies

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$\therefore$ It is a probability density fn

Mean & Standard deviation of Normal distribution

$$\text{Mean } (\mu) = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= \int_{-\infty}^{\infty} x \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}\, dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} x e^{-\frac{(x-\mu)^2}{2\sigma^2}}\, dx \quad - (1)$$

put $t^2 = \frac{(x-\mu)^2}{2\sigma^2}$

$$t = \frac{(x-\mu)}{\sqrt{2}\sigma}$$

$$\sqrt{2}\sigma t = x - \mu$$

$$x = \mu + \sqrt{2}\sigma t$$

D.w.r. to x

$$dx = 0 + \sqrt{2}\sigma . dt$$

$$dx = \sqrt{2}\sigma\, dt$$

$$(1) \Rightarrow \int_{-\infty}^{\infty} x f(x)\, dx = \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \int_{-\infty}^{\infty} x e^{-t^2} \sqrt{2}\cancel{\sigma}\, dt$$

$$= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} (\mu + \sqrt{2}\sigma t) e^{-t^2} . dt$$

$$= \frac{1}{\sqrt{\pi}} \left[\int_{-\infty}^{\infty} \mu e^{-t^2} dt + \sqrt{2}\sigma \int_{-\infty}^{\infty} t e^{-t^2} dt \right.$$

$$= \frac{\mu}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-t^2} dt + \frac{\sqrt{2}\sigma}{\sqrt{\pi}} \int_{-\infty}^{\infty} t e^{-t^2} dt$$

$$= \underbrace{\frac{\mu}{\sqrt{\pi}} \times 2 \int_{0}^{\infty} e^{-t^2} dt}_{\text{even function}} + \sigma \frac{\sqrt{2}}{\sqrt{\pi}} \underbrace{\int_{-\infty}^{\infty} t e^{-t^2} dt}_{\text{odd function}}$$

$$= \frac{2\mu}{\sqrt{\pi}} \int_{0}^{\infty} e^{-t^2} dt + 0$$

$$= \frac{\cancel{2}\mu}{\cancel{\sqrt{\pi}}} \times \frac{\cancel{\sqrt{\pi}}}{\cancel{2}}$$

$\therefore$ mean $= \mu$

$$\text{Variance}(\sigma^2) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx$$

$$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

put $t^2 = \frac{(x-\mu)^2}{2\sigma^2}$

$$t = \frac{x-\mu}{\sqrt{2}\sigma}$$

$$\sqrt{2}\sigma t = x-\mu$$

$$x = \mu + \sqrt{2}\sigma t$$

$$dx = \sqrt{2}\sigma\, dt$$

t also vary from $-\infty$ to ∞

(2)

$$= \int_{-\infty}^{\infty} 2t^2\sigma^2 \frac{1}{\cancel{\sigma}\sqrt{2\pi}}\, e^{-t^2}\, \sqrt{2}\cancel{\sigma}\, dt$$

$$= 2\sigma^2 \int_{-\infty}^{\infty} t^2 \frac{1}{\sqrt{\pi}}\, e^{-t^2}\, dt$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \int_{-\infty}^{\infty} t^2 e^{-t^2}\, dt$$

↳ even function

$$= \frac{2\sigma^2}{\sqrt{\pi}} \times 2\int_{0}^{\infty} t^2 e^{-t^2}\, dt$$

$f(t) = t^2 e^{-t^2}$

$f(-t) = +f(t)$

even function

Variance $= \frac{2\sigma^2}{\sqrt{\pi}} \times \int_{0}^{\infty} t\,(2t e^{-t^2})\, dt$

$u = t \qquad v = 2t e^{-t^2}$

W.K.T $\int u v\, dt = u\int v\, dt - \iint v.dt\; u'\, dt$

$$\int v.dt = \int 2t e^{-t^2} = \cancel{2t}\,\frac{e^{-t^2}}{-\cancel{2t}} = -e^{-t^2}$$

Variance $= \frac{2\sigma^2}{\sqrt{\pi}} \left\{ \left[t\,(-e^{-t^2}) \right]_{0}^{\infty} - \int_{0}^{\infty} -(e^{-t^2})\, 1.dt \right\}$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \left\{ 0 - 0 + \int_{0}^{\infty} e^{-t^2}.\, dt \right\}$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \int_{0}^{\infty} e^{-t^2}\, dt$$

$$= \frac{\cancel{2}\sigma^2}{\cancel{\sqrt{\pi}}} \times \frac{\cancel{\sqrt{\pi}}}{\cancel{2}}$$

$$= \sigma^2$$

Variance $= \sigma^2$

hence we can say that variance / S.D of normal distribution is equal to the variance / S.D of the given distribution

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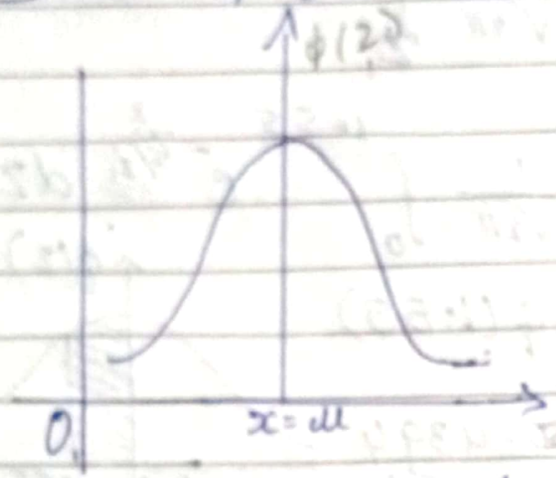
NOTE ①: The graph of the probability function $f(x)$ is a bell shaped curve symmetrical about the line $x = \mu$ & is called normal probability curve. The shape of the curve is



The line $x = \mu$ divides the total area under the curve which is equal to 1 into two equal parts. The area to the right as well as to the left of the line $x = \mu$ is 0.5

NOTE ②: $\phi(z) = \frac{1}{\sqrt{2\pi}} \int_0^z e^{-z^2/2}\, dz$

represents the area under standard normal curve from 0 to Z.

① $\int_{-\infty}^{\infty} \phi(z)\, dz = 1$

② $\int_{-\infty}^{0} \phi(z)\, dz = \int_{0}^{\infty} \phi(z)\, dz = 1/2$

③ $P(-\infty \le z \le \infty)$ or $P(z \geq 0) = 1/2$

also $P(-\infty < z < z_1) = P(-\infty < z \le 0) + P(0 \le z < z_1)$

④ $P(z < z_1) = 0.5 + \phi(z_1)$

⑤ also $P(z > z_2) = P(z \geq 0) - P(0 \le z < z_2)$

i.e $P(z > z_2) = 0.5 - \phi(z_2)$

problems

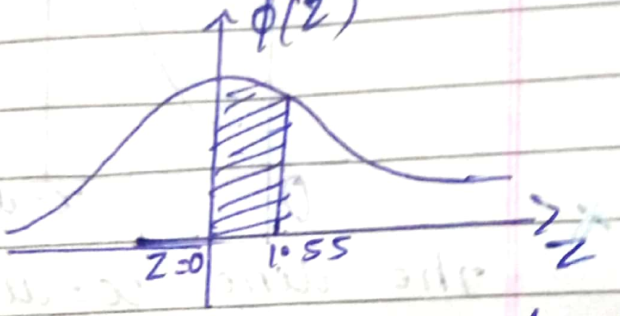
① Find the area under the standard normal curve b/w $Z=0$ & 1.55

Sol^{n}: $= \frac{1}{\sqrt{2\pi}} \int_{Z_1}^{Z_2} e^{-Z^2/2} \, dZ$

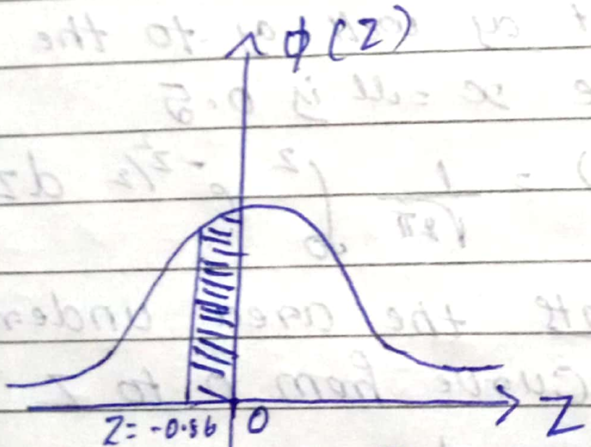
$= \frac{1}{\sqrt{2\pi}} \int_{0}^{1.55} e^{-Z^2/2} \, dZ$

$= \phi(1.55)$

$= 0.4394$



② Find the area under the standard normal curve between $Z=-0.86$ & $Z=0$



area $= \frac{1}{\sqrt{2\pi}} \int_{Z_1}^{Z_2} e^{-Z^2/2} \, dZ$

$= \frac{1}{\sqrt{2\pi}} \int_{-0.86}^{0} e^{-Z^2/2} \, dZ$

$= \frac{1}{\sqrt{2\pi}} \int_{0}^{0.86} e^{-Z^2/2} \, dZ$

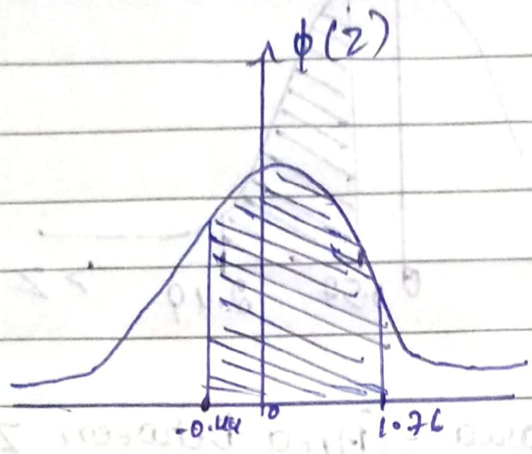
$= \phi(0.86)$

(8)

$P(-0.86 \le Z \le 0) = 0.30511$

(3) find the area of Standard normal curve between $Z = -0.44$ & $Z = 1.76$

Sol:



$$Area = \frac{1}{\sqrt{2\pi}} \int_{z_1}^{z_2} e^{-z^2/2}\, dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-0.44}^{1.76} e^{-z^2/2}\, dz$$

$$= \frac{1}{\sqrt{2\pi}} \left[\int_{-0.44}^{0} e^{-z^2/2}\, dz + \int_{0}^{1.76} e^{-z^2/2}\, dz \right]$$

$$= \frac{1}{\sqrt{2\pi}} \int_{0}^{0.44} e^{-z^2/2}\, dz + \frac{1}{\sqrt{2\pi}} \int_{0}^{1.76} e^{-z^2/2}\, dz$$

$$= \phi(0.44) + \phi(1.76)$$

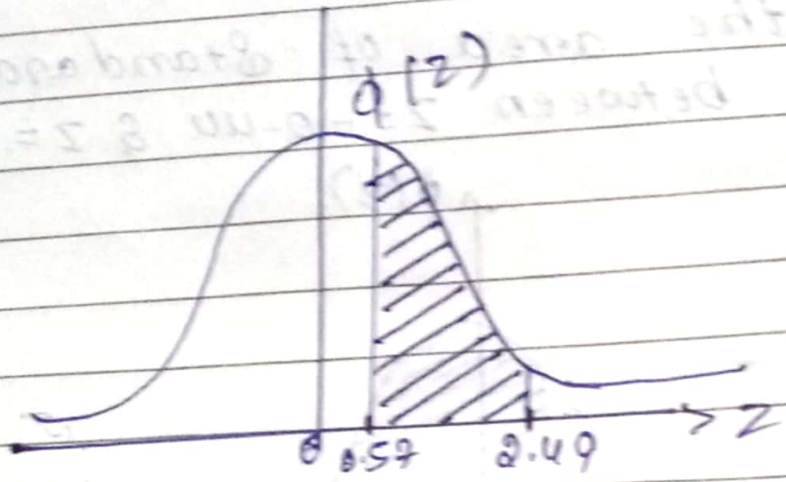
$$= 0.17003 + 0.4608$$

$$= 0.63083$$

Do yourself

(4) $Z = 0.57$ to 2.49

Sol^n:



Required area = (Area between $Z = 0$ to 2.49) − (Area b/w $Z = 0$ to 0.57)

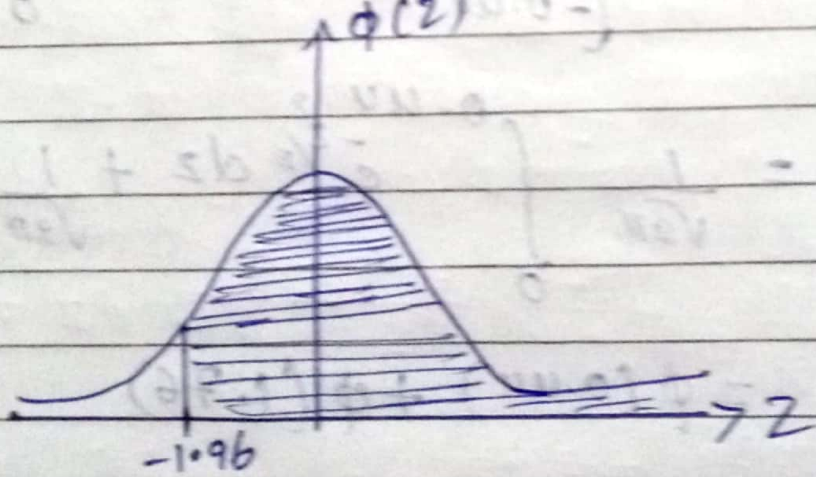
$= \phi(2.49) - \phi(0.57)$

$= 0.4936 - 0.2157$

$= 0.2779$ //

(5) Find the area of Standard normal curve to right of $Z = -1.96$

Sol^n:



Required Area = (Area between $Z = -1.96$ to 0) + (Area to right of $Z = 0$)

$= (\text{Area b/w } z = 0 \text{ to } 1.96) + 0.5$ by symmetry

$= \phi(1.96) + 0.5$

$= 0.4750 + 0.5$

$= 0.9750$

$P(Z \geq -1.96) = \underline{\underline{0.9750}}$

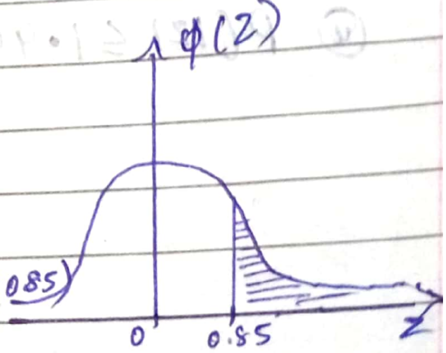
⑥ Evaluate the following probabilities with the help of normal probability tably.

① $P(Z \geq 0.85)$

② $P(-1.64 \leq Z \leq -0.88)$

③ $P(Z \leq -2.43)$

④ $P(|Z| \leq 1.94)$



Soln: ① $P(Z \geq 0.85)$

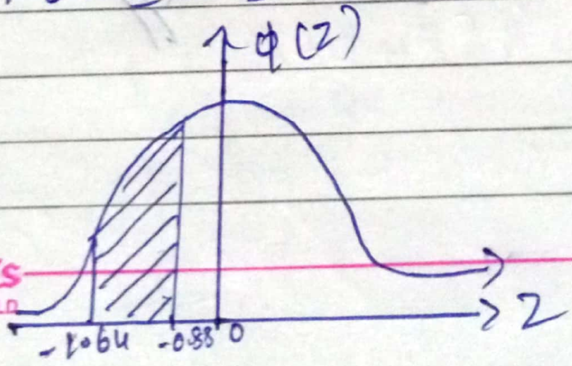
$P(Z \geq 0.85) = P(Z \geq 0) - P(0 \leq Z \leq 0.85)$

$= 0.5 - \phi(0.85)$

$= 0.5 - 0.30234$

$= 0.1977$

② $P(-1.64 \leq Z \leq -0.88)$



$P(-1.64 \le z \le -0.88) = P(0.88 \le z \le 1.64)$

$= P(0 \le z \le 1.64) - P(0 \le z \le 0.88)$

$= \phi(1.64) - \phi(0.88)$

$= 0.4495 - 0.31057$

$= 0.1389$ //

③ $P(z \le -2.43) = P(z \geq 2.43)$

$= P(z \geq 0) - P(z \le 2.43)$

$= 0.5 - \phi(2.43)$

$= 0.5 - 0.4925$

$= 0.0075$ //

④ $P(|z| \le 1.94) = P(-1.94 \le z \le 1.94)$

$= P(-1.94 \le z \le 0) + P(0 \le z \le 1.94)$

$= P(0 \le z \le 1.94) + P(0 \le z \le 1.94)$

$= \phi(1.94) + \phi(1.94)$

$= 2\phi(1.94)$

$= 2 \times 0.47381$

$= 0.94762$ //

(8) If x is a normal variate with mean 30 & standard deviation 5 find the probability that
(1) $26 \le x \le 40$ (2) $x \ge 45$

Sol: we have standard normal variate

$$Z = \frac{x-\mu}{\sigma} = \frac{x-30}{5}$$

(1) To find $P(26 \le x \le 40)$

If $x = 26$ $Z = -0.8$; If $x = 40, Z = 2$

$\therefore P(-0.8 \le Z \le 2) = P(-0.8 \le Z \le 0) + P(0 \le Z \le 2)$

$= P(0 \le Z \le 0.8) + P(0 \le Z \le 2)$

$= \phi(0.8) + \phi(2)$

$= 0.28814 + 0.47725$

$= 0.76539$

(2) $P(x \ge 45)$

If $x = 45$, $Z = 3$ hence we have to find $P(Z \ge 3)$

$P(Z \ge 3) = P(Z \ge 0) - P(Z \le 3)$ or $P(0 \le Z \le 3)$

$= 0.5 - \phi(3)$

$= 0.5 - 0.4987$

$= 0.0013$

Do yourself

① If x is normally distributed with mean 18 & S.D 4, find the following
① $P(x \geq 20)$ ② $P(x \leq 20)$

④ The marks of 1000 students in an examination follows a normal distribution with mean 70 & S.D 5. Find the no. of students whose marks will be
① less than 65 ② more than 75 ③ b/w 65 & 75

Soln: let x represents the marks of students

By data $\mu = 70$, $\sigma = 5$ $z = \frac{x-\mu}{\sigma}$

$z = \frac{x-70}{5}$

① If $x = 65$, $z = -1$

we have to find $P(z < -1)$

$P(z < -1) = P(z > 1)$

$= P(z \geq 0) - P(0 < z < 1)$

$= 0.5 - \phi(1)$

$= 0.5 - 0.3413$

$= 0.1587$

No. of students scoring less than 65 marks

$= 1000 \times 0.1587$

$= 158.7$

≈ 159

(ii) If $x = 75$, $z = 1$, we have to find $P(z > 1)$

$$P(z>1) = P(z \geq 0) - P(0 < z < 1)$$
$$= 0.5 - \phi(1)$$
$$= 0.5 - 0.3413$$
$$= 0.1587$$

∴ No/. of students scoring morethan 75 marks $= 1000 \times 0.1587 = 158.7 \simeq 159$

(iii) we have to find $P(65 < x < 75)$

If $x = 65$ $z = -1$, If $x = 75$, $z = 1$

hence

$$P(-1 < z < 1) = P(-1 < z < 0) + P(0 < z \leq 1)$$
$$= P(0 < z < +1) + P(0 < z < 1)$$
$$= 2P(0 < z < 1)$$
$$= 2\phi(1)$$
$$= 2(0.3413)$$
$$= 0.6826$$

No/. of students scoring marks b/w 65 & 75 $= 1000 \times 0.6826$
$= 682.6$
$\simeq 683$

(2) In a test on electric bulbs, it was found that the life time of a particular brand was distributed normally with an

average life of 2000 hrs & S.D of 60 hrs
If a firm purchases 2500 bulbs find
the no. of bulbs that are likely
to last for ① morethan 2100 hrs
② Lessthan 1950 hrs
③ b/w 1900 to 2100 hrs

Soln: By data $\mu = 2000$, $\sigma = 60$.

S.N.V. $Z = \frac{x-\mu}{\sigma} = \frac{x-2000}{60}$

① To find $P(x > 2100)$

If $x = 2100$ $Z = \frac{2100-2000}{60} = \frac{100}{60} = 1.67$

$P(x > 2100) = P(Z > 1.67)$
$= P(Z \geq 0) - P(0 < Z < 1.67)$
$= 0.5 - \phi(1.67)$
$= 0.5 - 0.4525$
$= 0.0475$

No. of bulbs that are likely to last for morethan 2100 hrs is 2500×0.0475
$= 118.75 \approx 119$

② To find $P(x < 1950)$

If $x = 1950$, $Z = -5/6 = -0.83$

$P(x < 1950) = P(Z < -0.83)$
$= P(Z > 0.83)$
$= P(Z \geq 0) - P(0 < Z < 0.83)$
$= 0.5 - \phi(0.83)$
$= 0.5 - 0.2967$
$= 0.2033$

ARUN'S GOLD

No. of bulbs that are likely to last for less than 1950 hrs is 2500×0.2033

$= 508.25$

≈ 508

③ To find $P(1900 < x < 2100)$

If $x = 1900$, $Z = -1.67$ & if $x = 2100$

$Z = 1.67$

$P(1900 < x < 2100) = P(-1.67 < Z < 1.67)$

$= P(-1.67 < Z < 0) + P(0 < Z < 1.67)$

$= P(0 < Z < 1.67) + P(0 < Z < 1.67)$

$= \phi(1.67) + \phi(1.67)$

$= 2\phi(1.67)$

$= 2 \times 0.45254$

$= 0.90508$

No. of bulbs that are likely to last between 1900 & 2100 hrs

$= 2500 \times 0.90508$

$= 2262.7$

≈ 2263